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Trajectory and focal length of circular Airy beams in linear index potentials

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ABSTRACT

In this paper, we investigate the propagation of abruptly autofocusing circular Airy beams (CAB) in linear index potential both analytically and numerically. Based on the initial field located far away from the center, we get the approximate analytical solutions to describe the trajectory and focal length of CAB in linear index potential. The modified focal length formula agrees well with numerical results for a wide range of the initial radius of CAB. Furthermore, the approximate method is also applicable to some dynamic potential varying along the propagation coordinate.

1. Introduction

In 1979, Berry and Balazs found a nonspreading solution of the potential-free Schrodinger equation: the Airy wave packet [1]. Using the concept of Airy wave packet in the context of quantum mechanics, Siviloglou et al. introduced theoretically and demonstrated experimentally the existence of the finite energy asymmetric Airy waves in the area of optics in 2007 [2,3]. These beams exhibit unique properties in propagating: non-spreading, self-healing and self-accelerating along a parabolic trajectory in free space which is similar to a projectile moving under the influence of gravity [4–7]. Due to these peculiar characteristics, self-accelerating Airy beams have been a hot topic in the past decade, but also have potential applications in many areas of physics, such as microscopy [8], lasing [9], micromanipulation of particles [10], optical routing [11], electron Airy waves [12], and laser micromachining of curved surfaces [13], etc.

Lately, abruptly autofocusing waves based on Airy packets have been proposed theoretically and observed experimentally without utilizing any lenses or nonlinearities [14–16]. These waves can be generated through the use of radially symmetric circular Airy waves or by appropriately superimposing one dimensional Airy wave packets. They can abruptly focus its energy right before a target while maintaining a low intensity profile until that very point. Meanwhile, many other abruptly autofocusing beams have also been proposed [17–19]. The abruptly autofocusing property makes the wave be an ideal candidate in biomedical treatment or micromachining with lasers since the abruptly autofocusing wave should only affect the intended area while leaving any preceding material intact. On the other hand, the modulation of abruptly autofocusing property is much important in many applications, and it has stirred a wide interest. Several strategies have been proposed to control the autofocusing dynamics, such as adding different optical vortices [20], blocking front light rings [21], and introducing a cone angle [22], etc. In addition, similar to traditional beams, the dynamics of abruptly autofocusing waves

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can be manipulated in the presence of an external potential. Up to now, the propagation dynamics of abruptly autofocusing wave composed of the appropriately superimposing Airy wave packets in linear potential has been discussed [23,24].

In this paper, the propagation of abruptly autofocusing CAB in linear index potential was analyzed. Approximate analytical solutions are derived to predict the trajectory and focal position and the solutions are valid for a wide range of the initial radius of CAB. The formulae of the focal length and trajectory can help us more intuitively understand the autofocusing behaviors of CAB in linear index potential. By properly designing the refractive-index gradient, the enhancement, reduction, and complete suppression of the autofocusing effect can be realized. What's more, the dynamic potential varying along the propagation direction is also discussed using the same approximate method.

2. Basic theory

The propagation dynamics of abruptly autofocusing CAB in graded index media ($\Delta n = -\delta_n \sqrt{x^2+y^2}$ with δ_n being a constant) is governed by the following paraxial wave equation:

$$i\frac{\partial A}{\partial z} + \frac{1}{2k}\nabla_\perp^2 A + k\frac{\Delta n}{n}A = 0, \tag{1}$$

where A is the slowly varying envelope of the beam, $\nabla_\perp^2 = \partial_x^2 + \partial_y^2$ is the transverse Laplacian operator, k is the wavenumber in the medium, n is the refractive index of the uniform medium, and $\Delta n << n$ indicates the graded index change. Here we introduce the dimensionless quantities $Z = z/kw_0^2, X = x/w_0, Y = y/w_0$, where w_0 is an arbitrary scaling constant, so that the paraxial wave equation can be written as follows:

$$i\frac{\partial u}{\partial Z} + \frac{1}{2}\nabla_{XY}^2 u - V(X,Y)u = 0, \tag{2}$$

where $\nabla_{XY}^2 = \partial_X^2 + \partial_Y^2$, $V(X,Y) = p\sqrt{X^2+Y^2}$ denotes the transverse linear potential, and $p = k^2 w_0^3 \delta_n / n$ indicates the gradient value of the linear index potential. For a symmetrical geometry, Eq. (2) transforms into

$$i\frac{\partial u}{\partial Z} + \frac{1}{2}(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}) - pru = 0, \tag{3}$$

where $r = \sqrt{X^2+Y^2}$ is the radial coordinate. Assuming the input beam is an inward CAB

$$u(r,0) = Ai(r_0 - r)\exp[a(r_0 - r)], \tag{4}$$

where $Ai(\bullet)$ denotes the Airy function, a is the decay parameter that makes the wave convey the finite energy, and r_0 stands for the initial radius of the main ring. For the inner of the main ring, the CAB decays exponentially, whereas the slowly decaying oscillations of the Airy tails occur outside this region. Because of the mathematical complication, it is hard to find the precise analytical solutions of Eqs. (3) and (4). In fact, if the initial radius of the main ring is large enough, almost all energy is essentially far away from the center during the initial stages of acceleration. As a result, we can adopt the approximation $\partial^2 u/\partial r^2 + \partial u/r\partial r \approx \partial^2 u/\partial r^2$. Thus, the third term on the left-hand side of Eq. (3) can be neglected. Consequently, Eq. (3) becomes

$$i\frac{\partial u}{\partial Z} + \frac{1}{2}\frac{\partial^2 u}{\partial r^2} - pru = 0, \tag{5}$$

which is similar to one-dimensional problem. In the computation of the above equation, we extended the variable r from zero to minus infinity. Note that this approach is reasonable, even for relatively small ring, due to the exponential decay of the inner of the main ring. To solve Eq. (5) with the initial condition given by Eq. (4), we introduce a new variable $r' = r_0 - r$, thus Eq. (5) can be expressed in the form

$$i\frac{\partial u}{\partial Z} + \frac{1}{2}\frac{\partial^2 u}{\partial r'^2} - p(r_0 - r')u = 0. \tag{6}$$

By introducing a trial solution of the form $u(r',Z) = \varphi(r',Z)\exp(-ipr_0 Z)$, Eq. (6) can be simplified as follows

$$i\frac{\partial \varphi}{\partial Z} + \frac{1}{2}\frac{\partial^2 \varphi}{\partial r'^2} + pr'\varphi = 0. \tag{7}$$

For an initial truncated Airy distribution $\varphi(r',0) = Ai(r')\exp(ar')$, Eq. (7) has a known solution [25,26]

$$\begin{aligned}\varphi(r',Z) = {} & Ai[r' - \frac{1}{4}(1+2p)Z^2 + iaZ] \times \exp[a(r' - \frac{1}{2}pZ^2 - \frac{1}{2}Z^2) \\ & + i(-\frac{1}{6}p^2Z^3 + pr'Z - \frac{1}{4}pZ^3 - \frac{1}{12}Z^3 + \frac{1}{2}a^2Z + \frac{1}{2}r'Z)]\end{aligned} \tag{8}$$