Optik 164 (2018) 84–92

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Original research article

Mitigating Internet bottleneck with fractional temporal evolution of optical solitons having quadratic–cubic nonlinearity

Hadi Rezazadeh[a], M.S. Osman[b], Mostafa Eslami[c], Mehmet Ekici[d], Abdullah Sonmezoglu[d], Mir Asma[e], W.A.M. Othman[e], B.R. Wong[e], Mohammad Mirzazadeh[f], Qin Zhou[g,*], Anjan Biswas[h,i,j], Milivoj Belic[k]

[a] Faculty of Modern Technologies Engineering, Amol University of Special Modern Technologies, Amol, Iran
[b] Department of Mathematics, Faculty of Science, Cairo University, Giza, Egypt
[c] Department of Mathematics, Faculty of Mathematical Sciences, University of Mazandaran, Babolsar, Iran
[d] Department of Mathematics, Faculty of Science and Arts, Bozok University, 66100 Yozgat, Turkey
[e] Institute of Mathematical Sciences, Faculty of Science, University of Malaya, 50603 Kuala Lumpur, Malaysia
[f] Department of Engineering Sciences, Faculty of Technology and Engineering, East of Guilan, University of Guilan, P.C. 44891-63157, Rudsar-Vajargah, Iran
[g] School of Electronics and Information Engineering, Wuhan Donghu University, Wuhan 430212, People's Republic of China
[h] Department of Physics, Chemistry and Mathematics, Alabama A&M University, Normal, AL 35762, USA
[i] Department of Mathematics and Statistics, College of Science, Al-Imam Mohammad Ibn Saud Islamic University, Riyadh 13318, Saudi Arabia
[j] Department of Mathematics and Statistics, Tshwane University of Technology, Pretoria 0008, South Africa
[k] Science Program, Texas A&M University at Qatar, PO Box 23874, Doha, Qatar

ARTICLE INFO

Article history:
Received 31 January 2018
Accepted 4 March 2018

OCIS:
060.2310
060.4510
060.5530
190.3270
190.4370

Keywords:
Solitons
Integrability
Perturbation
Conformable fractional derivative

ABSTRACT

This paper studies fractional temporal evolution of optical solitons with quadratic–cubic nonlinearity that comes with a few perturbation terms. Khalil's conformable fractional derivative as well as Liu's extended trial function scheme are applied to retrieve these soliton solutions. The results are applicable to mitigate Internet bottleneck that is a growing problem in telecommunications industry.

1. Introduction

Optical soliton perturbation with fractional temporal evolution is one of the viable means to address a growing problem in telecommunication industry, namely the Internet bottleneck. This problem leads to slow Internet traffic and eventually

* Corresponding author.
E-mail address: qinzhou@whu.edu.cn (Q. Zhou).

https://doi.org/10.1016/j.ijleo.2018.03.006
0030-4026/

blockage of the traffic. Several mechanisms have been proposed to address this concern. One of them is to choose time-dependent coefficients of dispersion and nonlinearity. A better and much more efficient method is to consider fractional temporal evolution which is what this paper will address.

The current work is to study the governing nonlinear Schrödinger's equation (NLSE) with quadratic–cubic (QC) nonlinearity in presence of perturbation terms and higher order dispersions such as third order dispersion (3OD) and fourth order dispersion (4OD). It is well known that NLSE as well as other forms of nonlinear evolution equations are addressed by a variety of mathematical methods [1–30]. In this paper, two approaches will reveal soliton solutions that will illustrate slow progress of solitons through optical fibers and other waveguides.

There are a variety of ways to define fractional temporal evolution such as Caputo derivative, Baleanu derivative and others. This paper will employ Khalil's conformable fractional derivative that will be first introduced. With this definition of temporal evolution, Kudryashov's method will lead to the soliton solutions of the perturbed NLSE. Finally, a second method that is known as Liu's extended trial equation method also will reveal additional forms of soliton solutions to the model. The rest of the paper is devoted to revisitation of these concepts and derivation of the soliton solutions.

1.1. Mathematical model

The governing resonant NLSE with conformable time fractional and perturbation terms that is studied in nonlinear optics is given in its dimensionless form as [7,8,28]

$$iD_t^{\alpha}q + aq_{xx} + \left(b_1\,|q| + b_2|q|^2\right)q = i\left\{\delta q_x - \gamma q_{xxx} - i\sigma q_{xxxx} + \lambda\left(|q|^2 q\right)_x + \theta\left(|q|^2\right)_x q\right\}, \tag{1}$$

where D_t^{α} is the conformable derivative operator of order $\alpha \in (0, 1]$ in the t-direction. In Eq. (1), the independent variables x and t respectively represent spatial and temporal variables. The dependent variable $q(x, t)$ gives the complex-valued wave profile together with $i = \sqrt{-1}$. The coefficient of the real-valued constant a is known as the group velocity dispersion. The nonlinear terms are given by the coefficients of b_1 and b_2, which represent quadratic and cubic forms, respectively. On the right hand side δ is the coefficient of inter-modal dispersion which appears when the group velocity of light propagating through multi-mode optical fibers or other optical waveguides depends on the optical frequency as well as the propagation mode involved. The coefficients of γ and σ are 3OD and 4OD, respectively. These appear when GVD is negligably small and thus the higher-order dispersions creep in order to maintain the necessary balance between dispersion and nonlinearity for the sustainment of optical solitons. The coefficient of λ is due to self steepening that is included to eliminate formation of shock waves. Finally, θ represents the coefficient of nonlinear dispersion.

1.2. Khalil's conformable fractional derivative

Recently, Khalil et al. introduced a new simple well-behaved definition of the fractional derivative called conformable derivative. The conformable fractional derivative of order is defined by the following definition [22].

Definition: Let $f: [0, \infty) \to R$, then, the conformable derivative of f of order α is defined as

$$D_t^{\alpha}(f)(t) = \lim_{\varepsilon \to 0} \frac{f(t + \varepsilon t^{1-\alpha}) - f(t)}{\varepsilon}, \tag{2}$$

for all $t > 0$, $\alpha \in (0, 1]$.

This new definition satisfies the properties in the following theorem:

Theorem 1. *Let $\alpha \in (0, 1]$, f, g be α-differentiable at a point t, then* [22]*:*

(i) $D_t^{\alpha}(af + bg) = aD_t^{\alpha}(f) + bD_t^{\alpha}(g)$, *for all* $a, b \in R$.
(ii) $D_t^{\alpha}(t^{\mu}) = \mu t^{\mu-\alpha}$, *for all* $\mu \in R$.
(iii) $D_t^{\alpha}(fg) = fD_t^{\alpha}(g) + gD_t^{\alpha}(f)$.
(iv) $D_t^{\alpha}\left(\frac{f}{g}\right) = \frac{gD_t^{\alpha}(f) - fD_t^{\alpha}(g)}{g^2}$.

If, in addition, f is differentiable, then $D_t^{\alpha}(f)(t) = t^{1-\alpha}\frac{df}{dt}$.

Abdeljawad [1] established the chain rule for conformable fractional derivatives as following theorem.

Theorem 2. *Let $f: (0, \infty) \to R$, be a function such that f is differentiable and also α-conformable differentiable. Let g be a differentiable function defined in the range of f. Then*

$$TD_t^{\alpha}(fog)(t) = t^{1-\alpha}g'(t)f'(g(t)), \tag{3}$$

where prime denotes the classical derivatives with respect to t.

Later on, many researchers established exact traveling wave solutions of various nonlinear fractional evolution equations via this fractional derivative [2,10,16,17,23,27,30].