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Analytic study on interactions of some types of solitary waves

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ABSTRACT

Interactions of solitary waves are widely used in such fields as nonlinear optics, plasma physics and theoretical physics. In this paper, interactions of some types of solitary waves, such as kink waves, are investigated analytically based on analytic solutions. Through theoretical analysis, interactions between solitary waves are presented. Various kinds of interactions are discussed by choosing different free parameters. Results are helpful to understand the nonlinear dynamics of solitary waves, and provide theoretical guidance for the design of all kinds of routing switches.

1. Introduction

With the further understanding and researches of science, researchers have found that nonlinear science has been related to every scientific fields [1–22]. As the significant branches of nonlinear science, solitons, chaos and fractals have been widely investigated [23–27]. These studies have not only theoretic meaning, but also practical worth [29–32]. Among them, scholars and researchers have come up with various research methods and meaningful results about nonlinear evolution equation, and a series of methods to construct the analytic soliton solutions, such as Darboux transform, Bäcklund transform and bilinear transformation, have been put forward [33–37].

On the other hand, symbolic computation is an interdisciplinary which contains mathematics, computer and artificial intelligence. It has become a symbol of modern scientific computing because of its ability to deal with algebraic calculus problems with a high-speed, highly accurate and systematic way. Symbolic computation in soliton theory started from the studies of partial differential equations in 1980s. As some constructive algorithms of nonlinear problems have been proposed, it has played a constructive role in the study of nonlinear evolution equations [38–44].

In this paper, we will study the $(3+1)$-dimensional potential-YTSF equations as follows [45],

$$4\frac{\partial^2 u}{\partial x\partial t}-\frac{\partial^4 u}{\partial x^3\partial z}-4\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x\partial z}-2\frac{\partial^2 u}{\partial x^2}\frac{\partial u}{\partial z}-3\frac{\partial^2 u}{\partial y^2}=0, \quad (1)$$

which can be used to describe the reacting mixtures and shallow water waves [46]. $u(x, y, z, t)$represents the slowly varying envelope while$t$is the normalized delay of time, and$x$,$y$and$z$refer to the normalized propagation distances. For Eq. (1), some types of analytic solutions, such as solitonic solutions, rational solutions, multi-soliton solutions, generalized

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solitary solutions, traveling wave solutions, periodic wave solutions and shock wave solutions, have been derived by different methods with symbolic computation [47–51].

However, to the best of our knowledge, there are few works about the interactions between kink waves for Eq. (1) [52]. Therefore, interactions between kink waves will be investigated in this paper. Various kinds of interactions will be analyzed by choosing the different parameters with numerical analysis. The structure of this paper will be as follows. In Section 2, the analytic solutions for Eq. (1) will be derived, and interactions between kink waves will be discussed. In Section 3, conclusions will be drawn.

2. Analytic solutions and discussions

The dependent variable transformation can be given as [45]

$$u(x, y, z, t) = 2[\ln f(x, y, z, t)]_x, \tag{2}$$

where$f(x, y, z, t)$is a real differentiable function respecting to x,y,z and t. Using the dependent variable transformation, the bilinear forms for Eq. (1) can be obtained as follows [45],

$$(4D_xD_t - D_x^3D_z - 3D_y^2)f \cdot f = 0, \tag{3}$$

$$(D_x^2 - \lambda D_xD_z)f \cdot f = 0. \tag{4}$$

Here,$\lambda$is a real constant.D_x,D_y,$D_z$and$D_t$are the bilinear operators [53–55]

$$D_z^mD_t^n(G \cdot F) = \left(\frac{\partial}{\partial z} - \frac{\partial}{\partial z'}\right)^m \left(\frac{\partial}{\partial t} - \frac{\partial}{\partial t'}\right)^n \times G(z, t)F(z', t')\Big|_{z'=z,t'=t} \tag{5}$$

Here, with m and n as non-negative integers. z' and t'as the independent variables, and $G(z, t)$ and $F(z, t)$ are the differentiable functions. Bilinear forms (3) and (4) can be solved by the following power series expansions for$f(x, y, z, t)$,

$$f(x, y, z, t) = 1 + \varepsilon f_1(x, y, z, t) + \varepsilon^2 f_2(x, y, z, t) + \varepsilon^3 f_3(x, y, z, t) + \cdots \tag{6}$$

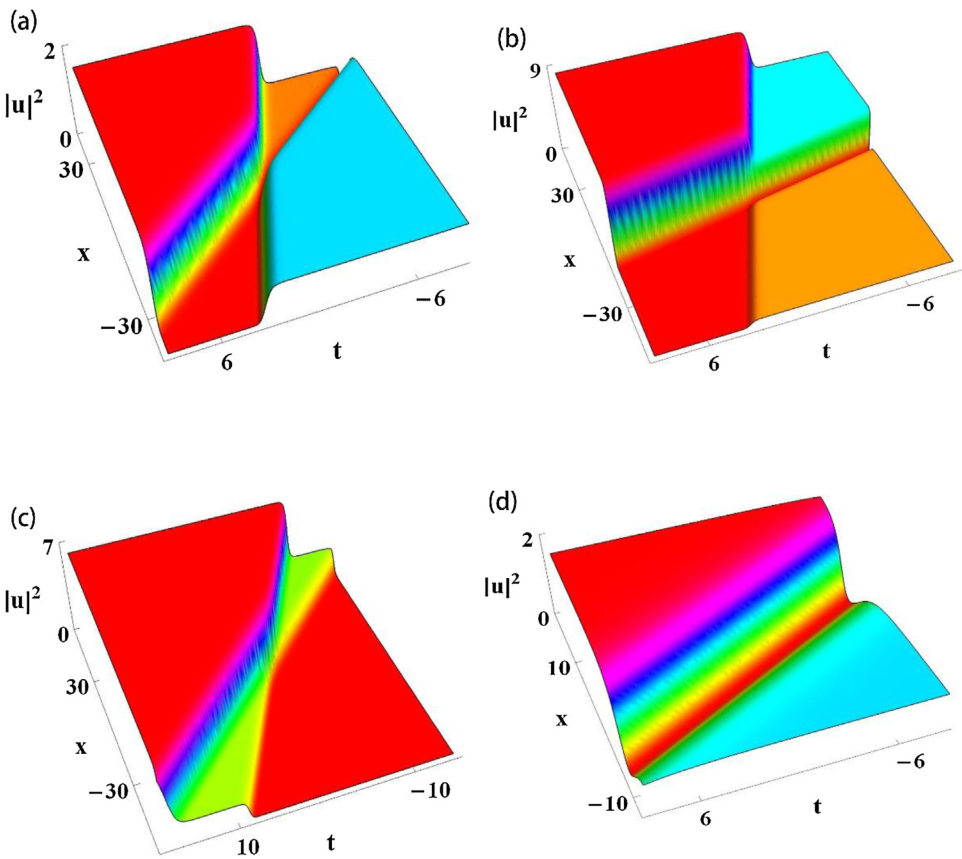

Fig. 1. Interactions of kink waves for solution (7). Parameters are $y = z = 0$,$\lambda = 0.97$,$\delta_1 = -1.1$,$\delta_2 = 0.63$: (a) $b_1 = 1.4$, $b_2 = -1.9$,$c_1 = 0.65$,$c_2 = -0.48$. (b) $b_1 = 1.4$, $b_2 = -1.9$,$c_1 = 1.5$,$c_2 = -0.44$. (c) $b_1 = 1.4$, $b_2 = -1.9$,$c_1 = 0.66$, $c_2 = 0.64$. (d) $b_1 = 0.66$, $b_2 = 0.47$,$c_1 = 0.65$,$c_2 = -0.48$.