Optik 164 (2018) 243–253

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Original research article

# New product and correlation theorems for the offset linear canonical transform and its applications

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ARTICLE INFO

*Article history:*
Received 27 January 2018
Accepted 27 February 2018

*Keywords:*
Convolution and product
Correlation
Offset linear canonical transform
Sampling
Filtering

ABSTRACT

The offset linear canonical transform (OLCT) plays an important role in optic signal processing. Many properties for this transform are already known, however, the product and correlation theorems doesn't have the simplicity and elegance comparable to that of the Fourier transform (FT). In this paper, we will introduce new product and correlation theorems for the OLCT, which have similar results for FT. First, the convolution and product theorem is introduced which states that a generalized convolution in the time domain is equivalent to simple multiplication operations for OLCT with a scaling factor. The new product structure does exactly parallel the product theorem for the FT. Moreover, the classical convolution and product theorem in the FT domain is shown to be the special case of our derived results. Furthermore, we propose a new correlation operation based on the convolution structure.Then, using the introduced convolution structure, we investigate the sampling theorem for the band-limited signal in the OLCT domain. We also discuss the applications of the new convolution for designing of multiplicative filter in the OLCT domain.



## 1. Introduction

The offset linear canonical transform (OLCT) [1–3], which is generalized version of the linear canonical transform (LCT) with a time-shifted and frequency-modulated [4–8]. Moreover, The OLCT encompasses a number of important signal processing operators and optical system modeling. Many operations, such as the Fourier transform (FT), the fractional Fourier transform (FRFT) [8–10], the Fresnel transform (FRST) [11], the offset FRFT [2], the LCT [4–8,12], time shifting and scaling, frequency modulation and others [3,13–18], are special case of the OLCT. With the progression of FRFT and LCT theory [19–39], OLCT also has evolved as an important tool in many fields in engineering [1–3,14–18]. For some signals processing, the FT, FRFT and LCT can not be applied directly, whereas the OLCT can be [1–3]. Therefore, it is worthwhile and interesting to understand and develop relevant theory for OLCT.

With intensive research of the OLCT, many properties have been found including phase shift, time shift, scaling, integration, differentiation and so on [1–3]. Simultaneously, the relevant theory of OLCT has been developed including the sampling theory, uncertainty principle, convolution theorem, and so on [3,13–18], which are generalizations of the corresponding properties of the FT, FRFT and LCT [19–39]. The OLCT has wide applications in optic and signal processing fields. However, the product and correlation theorems for OLCT [14] don't have the elegance and simplicity comparable to that of

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https://doi.org/10.1016/j.ijleo.2018.02.111
0030-4026/

**Table 1**
Some of the special case of the OLCT.

| Parameter $M$ | Corresponding transform |
|---|---|
| $M=(a, b, c, d, 0, 0)$ | Linear canonical transform (LCT) |
| $M=(\cos\theta, \sin\theta, -\sin\theta, \cos\theta, u_0, \omega_0)$ | Offset fractional Fourier transform (OFRFT) |
| $M=(\cos\theta, \sin\theta, -\sin\theta, \cos\theta, 0, 0)$ | Fractional Fourier transform (FRFT) |
| $M=(1, b, 0, 1, 0, 0)$ | Fresnel transform (FRST) |
| $M=(0, j, j, 0, 0, 0)$ | Laplace transform (LT) |
| $M=(0, 1, -1, 0, u_0, \omega_0)$ | Offset Fourier transform (OFT) |
| $M=(0, 1, -1, 0, 0, 0)$ | Fourier transform (FT) |
| $M=(1, 0, 0, 1, 0, \omega_0)$ | Frequency modulation |
| $M=(1, 0, 0, 1, u_0, 0)$ | Time shifting |
| $M=(d^{-1}, 0, 0, d, 0, 0)$ | Time scaling |

the FT. Hence, in order to strengthen the application of OLCT in signal processing, it is necessary to study its product and correlation theory.

Convolution theorem for a linear integral transform can be formulated in several ways. Xiang [14] introduced a product and correlation operation for the OLCT. However, the product and convolution structure derived in [14] also contains an extra chirp factor and hence does not exactly parallel the theorem given by FT. In addition, the OLCT of the correlation of the two functions is the product of their OLCTs and a phase factor. Due to an extra phase factor, it is difficult to implement in the engineering since it is nearly impossible to generate a chirp signal accurately. Recently, a generalized convolution theorem for the OLCT has been introduced based on generalized translation [18], which preserves the similar convolution structure of the FT. However, it is expressed by a triple integral. Compared to a single integral form as in the ordinary convolution expression, it is not easy to implement for filtering.

In this paper, we introduce new product and correlation structures for the OLCT that is different from those introduced in Refs. [14,18]. We introduce two new operation for the OLCT that works well with both the OLCT and its inverse. Those structures have the elegance and simplicity similar to FT. Moreover, it can be expressed by a one dimensional integral. The rest of the paper is organized as follows. In Section 2, we present a brief review of the OLCT, convolution theory and band-limited signals. In Section 3, we will introduce new product and correlation theorems. In Section 4, we will discuss two applications: the convolution for the designing of multiplicative filter and the reconstruction of the band-limited signals in the OLCT domain. Finally, in Section 5, we make a conclusion.

## 2. Preliminaries

### 2.1. Offset linear canonical transform (OLCT)

The OLCT of a function $f(t)$ is defined with six parameter $M=(a, b, c, d, u_0, \omega_0)$ as follows [3]

$$F_M(u)=L^M[f(t)](u)$$

$$=\begin{cases}\displaystyle\int_{-\infty}^{+\infty} f(t)K_M(u,t)dt,\ b\neq 0\\ \sqrt{d}e^{j\dfrac{cd}{2}(u-u_0)^2+j\omega_0 u}f[d(u-u_0)],\ b=0\end{cases} \tag{1}$$

where

$$K_M(u,t)=T_b e^{j\frac{1}{2b}\left[d(u_0^2+u^2)-2u(du_0-b\omega_0)+2t(u_0-u)+at^2\right]} \tag{2}$$

Where $T_b=\sqrt{1/(j2\pi b)}$, So, we will confine the OLCT for $b\neq 0$. Its inverse transformation can be defined as:

$$f(t)=L^{M^{-1}}[F_M(u)](t)=C\int_{-\infty}^{+\infty} F_M(u)K_{M^{-1}}(u,t)du \tag{3}$$

Where parameter$M^{-1}=(d, -b, -c, a, b\omega_0-du_0, cu_0-a\omega_0)$ and $C=e^{j\frac{1}{2}\left(cdu_0^2-2adu_0\omega_0+ab\omega_0^2\right)}$. Some special cases of the OLCT can be summarized in Table 1.

The OLCT has three basic properties as follows [2,3]:

Property 1: time shifting

$$L^M[f(t-\tau)](u)=F_M(u-a\tau)e^{-j\frac{ac\tau^2}{2}+jc\tau(u-u_0)+ja\tau\omega_0} \tag{4}$$