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Optical soliton perturbation with fractional temporal evolution by generalized Kudryashov's method

Anjan Biswas[a,b,c], Abdullah Sonmezoglu[d], Mehmet Ekici[d], Mohammad Mirzazadeh[e], Qin Zhou[f,*], Seithuti P. Moshokoa[c], Milivoj Belic[g]

[a] Department of Physics, Chemistry and Mathematics, Alabama A&M University, Normal, AL 35762, USA
[b] Department of Mathematics and Statistics, College of Science, Al-Imam Mohammad Ibn Saud Islamic University, Riyadh 13318, Saudi Arabia
[c] Department of Mathematics and Statistics, Tshwane University of Technology, Pretoria 0008, South Africa
[d] Department of Mathematics, Faculty of Science and Arts, Bozok University, 66100 Yozgat, Turkey
[e] Department of Engineering Sciences, Faculty of Technology and Engineering, East of Guilan, University of Guilan, P.C. 44891-63157, Rudsar-Vajargah, Iran
[f] School of Electronics and Information Engineering, Wuhan Donghu University, Wuhan 430212, People's Republic of China
[g] Science Program, Texas A&M University at Qatar, PO Box 23874, Doha, Qatar

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ABSTRACT

This paper retrieves optical soliton solutions with fractional temporal evolution by the aid of generalized Kudryashov's method. There are four types of nonlinear fibers that are studied here. Bright, dark and singular soliton solutions are retrieved. The existence criteria of these solitons are also presented.

1. Introduction

The dynamics of soliton propagation through optical fibers and several other waveguides is a very active area of research in telecommunications industry. There are several avenues in this topic that are consistently being addressed from various parts of the globe. They are integrability issue, perturbation theory, conservation laws, super-continuum generation and several others. This paper will address one such important topic. This is about soliton generation with fractional temporal evolution. This topic has generated considerable interest in the field of optics with a growing demand of broader bandwidth and addressing the problem of internet bottleneck. Thus, fractional evolution of pulses slows down its generation while the internet traffic can be controlled along the other direction. This technology has played a vital role in the growing demand

* Corresponding author.
E-mail address: qinzhou@whu.edu.cn (Q. Zhou).

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for internet across the globe. There are several mathematical methodologies to address this issue that have been considered during the past few years [1–10]. This paper will study the model, namely the perturbed nonlinear Schrödinger's equation, (NLSE) with fractional temporal evolution by the aid of generalized Kudryashov's approach. The integration scheme is introduced and subsequently applied to fractional NLSE that is studied with four forms of nonlinear fibers. The remainder of the paper details the analysis.

1.1. Generalized Kudryashov's method with conformable fractional derivatives

In this section, we recapitulate fractional generalized Kudryashov's method coupled with conformable fractional derivative for obtaining traveling wave solutions to fractional nonlinear evolution equations (NLEE) [1,6].

Remark. We denote $\partial^\alpha/\partial t^\alpha f(t)$ for ${}_tT_\alpha(f)(t)$, to represent temporal conformable fractional derivatives of f of order α.

Suppose a fractional NLEE for $u(x, t)$ takes the form

$$P\left(\frac{\partial^\alpha u}{\partial t^\alpha}, \frac{\partial u}{\partial x}, \frac{\partial^{2\alpha} u}{\partial t^{2\alpha}}, \frac{\partial^2 u}{\partial x^2}, \ldots\right) = 0, \qquad 0 < \alpha \le 1, \tag{1}$$

where P represents a polynomial in u and $\partial^\alpha u/\partial t^\alpha$ and $\partial^{2\alpha} u/\partial t^{2\alpha}$ are conformable fractional derivatives of $u(x, t)$. The algorithmic approach to generalized Kudryashov's method can be recounted in the following steps:

(2) *Step 1*: To locate traveling wave solutions of Eq. (1), we come up with the wave variable:

$$u(x, t) = U(\xi), \qquad \xi = kx - v\frac{t^\alpha}{\alpha}, \tag{2}$$

where k and v are constants which will be found afterwards.
Substituting Eq. (2) into Eq. (1), we locate the following ordinary differential equation (ODE):

$$Q(U, U', U'', \ldots) = 0. \tag{3}$$

(3) *Step 2*: Assume the solution of the Eq. (3) can be:

$$U(\xi) = \frac{\sum_{i=0}^{N} k_i Q^i(\xi)}{\sum_{j=0}^{M} l_j Q^j(\xi)} = \frac{A[Q(\xi)]}{B[Q(\xi)]}, \tag{4}$$

where k_i ($i = 0, a, \ldots, N$) and l_j ($j = 0, 1, \ldots, M$) are constants unknown at this stage, and $Q(\xi)$ is $1/(1 \pm e^\xi)$. We note that the function $Q(\xi)$ solves the following equation [2]

$$Q_\xi = Q^2 - Q. \tag{5}$$

Considering (4) along with (5), we recover

$$U'(\xi) = (Q^2 - Q)\left[\frac{A'B - AB'}{B^2}\right], \tag{6}$$

$$U''(\xi) = \frac{Q^2 - Q}{B^2}\left[(2Q - 1)(A'B - AB') + \frac{Q^2 - Q}{B}[B(A''B - AB'') - 2A'BB' + 2A(B')^2]\right], \tag{7}$$

and so on. Here, prime denotes the derivative with respect to ξ.

(4) *Step 3*: Based on homogeneous balance between highest order derivatives and nonlinear terms appearing in the ODE (3), we can determine a relation between M and N. We can next choose some values of M and N.

(5) *Step 4*: Substituting expressions from Eqs. (4)–(7) into Eq. (3), we arrive at a polynomial $\Lambda(Q)$ of Q. Upon setting the coefficients of this polynomial to zero, we obtain a system of algebraic equations. Solving this system, we can find the unknown parameters. This leads to the exact solutions to Eq. (1).

1.2. Governing model

The dimensionless form of the perturbed NLSE with fractional temporal evolution [4,5,7,8,10] is

$$i\frac{\partial^\alpha q}{\partial t^\alpha} + aq_{xx} + bF(|q|^2)q = i\{\delta q_x + \beta(|q|^{2n}q)_x + \nu(|q|^{2n})_x q\}, \qquad t > 0, \quad 0 < \alpha \le 1, \tag{8}$$

where x represents spatial variable along the fiber and t represents temporal variable, while a and b are real-valued constants. The dependent variable $q(x, t)$ is a complex-valued function. Next, δ, β and ν are real-valued constant coefficients. Here in (1), the first term represents the fractional temporal evolution, while the coefficient of a is the group velocity dispersion (GVD) and b stems from fiber nonlinearity. On the right hand side of (1), δ is the inter-modal dispersion that is considered in addition to chromatic dispersion. Finally, β and ν are the coefficients of self-steepening and nonlinear dispersion respectively. The