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Original research article

Chirped bright and dark solitons of electric and magnetic coupled nonlinear field equations in negative-index metamaterials

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ABSTRACT

The generalized coupled nonlinear Schrödinger equations govern the pulse propagation of electric and magnetic fields in negative index materials. Based on the F-function expansion method, Jacobian elliptic function solution and bright and dark soliton solutions are obtained. Dynamics behaviors of bright and dark solitons are studied. Bright and dark solitons exist both in the normal dispersion regimes. The amplitudes and widths of bright and dark solitons are decided by parameters S_E, S_H related to electric and magnetic self-steepening effects. The phase chirp of bright and dark soliton shifts along the positive time-axis with the increase of the propagation distance.

1. Introduction

The negative index metamaterials (NIMs) with negative electric and magnetic permeability [1–3], also called as left-handed metamaterials, exhibit rich and novel properties including the reversal of Doppler effect, the reversal of Cherenkov effect and the reversal of Snell's law, and so on [4]. Because these uncommon properties, NIMs become novel and exciting field of theoretical and experimental research after being theoretically introduced by Veselago [5] and experimentally realized by Pendry [6] and Smith et al. [7].

Optical solitons in different materials have been extensively investigated [8–11]. As one of man-made artificially structured materials, NIMs provide the flexibility of controlling optical pulses inside it at one's will. Therefore, the investigation of nonlinear pulse propagation, specifically optical solitons, is a hot topic of research. As the first significant attempt, Scalora et al. [12] obtained a new generalized nonlinear Schrodinger equation (NLSE) to describe ultrashort pulse propagation in bulk NIM exhibiting frequency-dependent dielectric susceptibility and magnetic permeability without considering nonlinear magnetization. Shen et al. [13] studied soliton and roguelike wave solutions in the transmission line analog of a nonlinear left-handed metamaterial. On the other hand, considering both nonlinear electric and magnetic polarization, Wen et al. [14] obtained a coupled NLSE for the propagation of few cycles pulse in an NIM. Sarma et al. [15] obtained a NLSE to describe pulse propagation in an NIM created from split-ring resonators and arrays of wires embedded in an electric and magnetic nonlinear Kerr medium.

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Optical solitons based on NLSE has been extensively studied to describe optical propagation in common media [16–19]. However, optical solitons in NIMs are relatively few discussed. In this paper, we focus on the generalized coupled NLSEs governing the pulse propagation of electric and magnetic fields in negative index materials. Based on the F-function expansion method, Jacobian elliptic function solution and bright and dark soliton solutions are obtained, and dynamics behaviors of bright and dark solitons are studied.

2. Jacobian elliptic function solution and soliton solutions

The coupled nonlinear field equations including both the electric and magnetic field envelopes for NIMs embedded into a Kerr medium is described by [15]

$$u_z = -\mathrm{i}\frac{\mathrm{sgn}(\beta_2)}{2}u_{tt} + \mathrm{i}(|u|^2 + |v|^2)u - S_E(|u|^2u)_t,$$

$$v_z = -\mathrm{i}\frac{\mathrm{sgn}(\beta_2)}{2}v_{tt} + \mathrm{i}(|u|^2 + |v|^2)v - S_H(|v|^2v)_t, \tag{1}$$

where $u(z, t)$ and $v(z, t)$ represent normalized complex electric and magnetic field envelopes respectively. $\mathrm{sgn}(\beta_2)$ denotes the sign of group velocity dispersion and equals ±1, $S_E \equiv |P_s|/T_0$ and $S_H \equiv |Q_s|/T_0$ describe electric and magnetic self-steepening effects respectively with electric and magnetic self-steepening coefficients P_s and Q_s and pulse width T_0. All terms depend upon electrical permittivity ϵ and magnetic permeability μ. The physical details of Eq. (1) can be found in Ref. [15]. Modulational instability analysis of Eq. (1) has been studied in Ref. [15]. When parameters S_E and S_H satisfy the condition $S_E = a^2S_H$ and field envelopes exist the constraint $v = au$ in Eq.(1), chirped soliton-like solutions have been obtained [20]. Our interest in this paper is to derive exact analytical solutions of Eq.(1) without restrictions on S_E and S_H.

In order to derive Jacobian elliptic function solution and soliton solutions, we use the ansatz

$$u(z, t) = A(\xi)\exp[\mathrm{i}\phi(\xi)],\ v(z, t) = B(\xi)\exp[\mathrm{i}\psi(\xi)], \tag{2}$$

where the amplitudes $A(\xi)$ and $B(\xi)$ and phases $\phi(\xi)$ and $\psi(\xi)$ with travelling frame of reference as $\xi = pz + qt$ with two constants p and q.

Inserting ansatz (2) into Eq. (1), and separating real and imaginary parts leads to

$$[p - \mathrm{sgn}(\beta_2)q^2\phi_\xi + 3qS_EA^2]A_\xi - \frac{1}{2}\mathrm{sgn}(\beta_2)q^2A\phi_{\xi\xi} = 0, \tag{3}$$

$$\frac{1}{2}\mathrm{sgn}(\beta_2)q^2A_{\xi\xi} + (qS_E\phi_\xi - 1)A^3 + \left[p\phi_\xi - \frac{1}{2}\mathrm{sgn}(\beta_2)q^2\phi_\xi^2 - B^2\right]A = 0, \tag{4}$$

$$[p - \mathrm{sgn}(\beta_2)q^2\psi_\xi + 3qS_HB^2]B_\xi - \frac{1}{2}\mathrm{sgn}(\beta_2)q^2B\psi_{\xi\xi} = 0, \tag{5}$$

$$\frac{1}{2}\mathrm{sgn}(\beta_2)q^2B_{\xi\xi} + (qS_H\psi_\xi - 1)B^3 + \left[p\psi_\xi - \frac{1}{2}\mathrm{sgn}(\beta_2)q^2\psi_\xi^2 - A^2\right]B = 0. \tag{6}$$

From Eqs. (3) and (5), we obtain

$$\phi_\xi = \frac{3S_E}{2q\mathrm{sgn}(\beta_2)}A^2 + \frac{p}{q^2\mathrm{sgn}(\beta_2)} + \frac{\phi_0}{A^2}, \tag{7}$$

and

$$\psi_\xi = \frac{3S_H}{2q\mathrm{sgn}(\beta_2)}B^2 + \frac{p}{q^2\mathrm{sgn}(\beta_2)} + \frac{\psi_0}{B^2}, \tag{8}$$

with two integral constants ϕ_0 and ψ_0. Here we choose $\phi_0 = 0$ and $\psi_0 = 0$.

Inserting Eqs. (7) and (8) into Eqs. (4) and (6), we derive

$$A_{\xi\xi} + d_1A^5 + e_1A^3 + f_1A = 0, \tag{9}$$

and

$$B_{\xi\xi} + d_2B^5 + e_2B^3 + f_2B = 0, \tag{10}$$

with $d_1 = 3S_E^2/[4(\mathrm{sgn}(\beta_2))^2q^2]$, $e_1 = 2pS_E/[(\mathrm{sgn}(\beta_2))^2q^3] - 2/[\mathrm{sgn}(\beta_2)q^2]$, $f_1 = p^2/[(\mathrm{sgn}(\beta_2))^2q^4] - 2B^2/[\mathrm{sgn}(\beta_2)q^2]$, $d_2 = 3S_H^2/[4(\mathrm{sgn}(\beta_2))^2q^2]$, $e_2 = 2pS_H/[(\mathrm{sgn}(\beta_2))^2q^3] - 2/[\mathrm{sgn}(\beta_2)q^2]$, $f_2 = p^2/[(\mathrm{sgn}(\beta_2))^2q^4] - 2A^2/[\mathrm{sgn}(\beta_2)q^2]$.

Via the F-function expansion method and homogenous balance principle [21,22], we can assume exact ansatz of Eqs. (9) and (10) as

$$A = \sqrt{a_0 + a_1F(\xi)}, \quad B = \sqrt{b_0 + b_1F(\xi)}, \tag{11}$$