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Gray optical soliton, linear stability analysis and conservation laws via multipliers to the cubic nonlinear Schrödinger equation

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A B S T R A C T

This paper addresses the cubic nonlinear Schrödinger equation with a bounded potential (CNLSE) which describes optical solitary waves propagation properties in optical fiber. A gray optical soliton solution of this equation is retrieved for the first time by adopting an appropriate solitary wave ansatz which play a vital role in understanding various physical phenomena in nonlinear systems. The integration lead to a constraint condition on the solitary wave parameters which must hold for the soliton to exist. We studied the conservation laws (Cls) of the CNLSE by analyzing a system of partial differential equations (PDEs) obtained by transforming the equation into real and imaginary components. The multiplier approach is employed to retrieve the conservation laws. Moreover, the modulation instability (MI) analysis of the model is studied by employing the linear-stability analysis and the MI gain spectrum is got. Physical interpretations of the acquired results are demonstrated. It is hoped that the results reported in this paper can enrich the nonlinear dynamical behaviors of the CNLSE.

1. Introduction

It is well known that NLSEs appear in a lot of areas of engineering sciences, physical and biological sciences. In particular, the NLSEs appears in fluid dynamics, nonlinear optics, plasma and nuclear physics [1–3]. Efficient ansatz techniques have been employed to investigate various types of solitary waves solutions with interesting properties to for NLSEs [4–7]. Triki et al. [8] recently proposed an ansatz for investigating black and gray optical solitary waves. During the last decades, a variety of techniques [5–56] have been recommended to derive the exact analytical solutions of nonlinear partial differential equations (NPDEs).

In this work, we aim to investigate the gray optical soliton solution to the CNLSE using an efficient envelope function ansatz [8] and thereafter retrieve the conservation laws inherited by the model using the method of multiplier [11]. Finally, using the linear stability analysis [2,12–15], the MI analysis aspect of the equation will be studied.

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2. Theoretical model

The CNLSE that will be studied in this paper is given by [9,10]:

$$ih\psi_t - \alpha\psi + m^{-1}h^2\frac{\psi_{xx}}{2} + \gamma\psi|\psi|^2 = 0. \tag{1}$$

For the model described by Eq. (1), the dependent variable is $\psi(x, t)$ that denoting the complex-valued profile. The independent variable x is the spatial variable, while t denotes the temporal variable. The constant γ tends to a finite constant, α is bounded, h is very small and m is any real constant [9]. The model describes the propagation properties of optical solitary wave solutions in optical fibers. It has been shown in previous time that the model includes standing wave solutions, which tend to the all non-degenerate critical points. A comprehensive study of the non spreading wave packets for Eq. (1) have also been reported in [9]. The optical solitary waves including the dark, bright and complexion solitary waves of Eq. (1) have been reported [10].

3. Gray optical soliton

To investigate the gray optical soliton solutions of Eq. (1) we consider a solitary wave ansatz solution of the form [8]:

$$\psi(x, t) = A(x, t) \times e^{i\phi(x,t)}, \tag{2}$$

where

$$\phi(x, t) = -kx + \omega t + \nu. \tag{3}$$

In Eq. (2), $A(x, t)$ is a complex function and ϕ represents the phase shift function. In Eq. (3), k represents the phase components representing the wave numbers and ω represents the parameter of frequency shift, respectively, while ν is the phase constant.

In order to investigate localized gray solutions of Eq. (1), we adopt the following ansatz solution [8]:

$$A(x, t) = \mathrm{i}\beta + \lambda\tanh[\eta(x - vt)], \tag{4}$$

where η and v are the pulse width and the inverse group velocity shift. It may be worth observing that we are investigating soliton solutions with non-zero amplitude as t approaches infinity, since the envelope function $A(x, t)$ is complex, the amplitude of $A(x, t)$ reads

$$|A(x, t)| = \left\{\lambda^2 + \beta^2 - \lambda^2\mathrm{sech}^2[\eta(x - vt)]\right\}^{1/2}, \tag{5}$$

and its corresponding nonlinear phase shift

$$\psi_{NL} = \arctan\left[\frac{\beta}{\lambda\tanh[\eta(x - vt)]}\right]. \tag{6}$$

In the event when $\beta = 0$ in Eq. (4), dark optical solitons have been called black optical solitons [8,9]. But when $\beta \neq 0$ in Eq. (4), then we have a gray optical soliton which is a type of black optical soliton with a complex amplitude parameter [8,9]. Both of these types of solitons play a vital role in understanding various physical phenomena in nonlinear systems. Substituting Eq. (4) into Eq. (1), one obtains:

$$A(h^2k^2 + 2m\alpha + 2hm\omega - 2m\gamma|A|^2) + ih(2hkA_x + ihA_{xx} - 2mA_t) = 0. \tag{7}$$

Putting Eq. (4) into Eq. (7), we get

$$\begin{aligned} &ih^2k^2\beta + 2im\alpha\beta - 2im\beta^3\gamma - 2im\beta\gamma\lambda^2 + 2ihm\beta\omega + 2ih^2k\eta\lambda\mathrm{sech}(\tau)^2 + \\ &2ihmv\eta\lambda\mathrm{sech}(\tau)^2 + 2im\beta\gamma\lambda^2\mathrm{sech}(\tau)^2 + h^2k^2\lambda\tanh(\tau) + 2m\alpha\lambda\tanh(\tau) - \\ &2m\beta^2\gamma\lambda\tanh(\tau) - 2m\gamma\lambda^3\tanh(\tau) + 2hm\lambda\omega\tanh(\tau) + 2h^2\eta^2\lambda\mathrm{sech}(\tau)^2 \\ &\tanh(\tau) + 2m\gamma\lambda^3\mathrm{sech}(\tau)^2\tanh(\tau) = 0, \end{aligned} \tag{8}$$

where $\tau = \eta(x - v\ t)$. Equating the summation of the coefficients of $\mathrm{sech}(\tau)$ and $\tanh(\tau)$ functions having the similar exponents to zero, we acquire:

Constants:

$$i\beta\left(h^2k^2 + 2m\left(\alpha - \gamma\left(\beta^2 + \lambda^2\right)\right) + 2hm\omega\right) = 0, \tag{9}$$

$\mathrm{sech}^2(\tau)$:

$$2i\lambda(h(hk + mv)\eta + m\beta\gamma\lambda) = 0, \tag{10}$$