Optik 164 (2018) 721–728

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Original research article

Analysis on spatio-temporal locality and degradation of the rogue wave in an optical fiber

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ARTICLE INFO

Article history:
Received 1 March 2018
Received in revised form 20 March 2018
Accepted 21 March 2018

Keywords:
Optical rogue wave
Spatio-temporal locality
Degradation
Rogue wave clusters

ABSTRACT

In this research, we give an analysis on the conditions and causes of breaking rogue wave's spatio-temporal locality and degradation of high-order rogue wave in an optical fiber by numerical simulation and analytic theory. Our simulation results show that both of increasing input power and third-order dispersion can undermine the spatio-temporal locality of the rogue wave. And they also cause the degradation of the high-order rogue wave as well. Then we find that the rogue wave clusters theory has an appropriate explanation for these phenomena. We consider that the essence of these phenomena is the interaction of rogue wave clusters on the spatio-temporal plane, in addition, give a detailed analysis of this by rogue wave triplet model. And the theoretical results are in good agreement with the simulation results in fiber.

1. Introduction

Rogue wave is a kind of pulse which has higher than a kite peak energy. It is widespread in nature especially in many nonlinear systems, such as ocean, optical fiber, plasma and BEC. Countless experiments confirmed the existence of this kind of super pulse [1–6]. Rogue wave has verities of definitions, while the rogue wave that we research all means rogue wave with rational solution. Without exception, the shape of such rogue wave solutions is one or more peaks on a plane wave background. The first rogue wave solution was constructed by a British mathematician Peregrine in 1982. This is the rogue wave with lowest order which was called Peregrine soliton [7]. Higher order rational solutions can be obtained by the modified Darboux transformation or the Wronskian determinant method [8,9], which has a more complex structure and a higher peak energy. However, whether it is Peregrine soliton or higher-order rogue wave, these rational solutions all have the same property calls spatio-temporal locality. It is shown as the rogue wave only exists within a finite range in the spatio-temporal plane (hereinafter referred to as (T, Z) plane). In other words, rogue wave is localized in both time and space rather than like Ahkmediev breathers only in the space direction or Kuznetsov-Ma solitons only in the time direction [10–16].

The rogue wave with ideal spatio-temporal locality exists only in the mathematical theory of rogue wave. Due to various factors, the rogue wave in reality doesn't always keep its spatio-temporal locality in any situation. For example, in an optical fiber, the rogue wave will have spatio-temporal locality only when the initial pulse satisfies particular conditions and all of the high-order effects in fiber are ignorable. When the initial parameters of the pulse change or the high-order effects in fiber become too obvious to ignore, the spatio-temporal locality of the rogue wave may be break. For example, the rogue wave can

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https://doi.org/10.1016/j.ijleo.2018.03.081
0030-4026/

split in fiber and this is a form of spatio-temporal locality breaking [17]. For the high-order rogue wave, the spatio-temporal locality breaking is often accompanied by the rogue wave degradation. It is shown as the peak energy of the high-order rogue wave declines rapidly, even the high-order rogue wave completely degradate into a series of Peregrine solitons. In this article, the conditions of spatio-temporal locality breaking and degradation of the high-order rogue wave in fiber are given by numerical simulation. And the causes of these phenomena are analyzed by the theory of rogue wave clusters.

2. Equation and rational solutions

The model of optical fiber we use is a typical single-mode fiber. The process of optical pulse evolution in a single-mode fiber is described by the generalized nonlinear Schrödinger equation [18]:

$$\frac{\partial A}{\partial z}+\frac{\alpha}{2}A+\sum_{k\geq 2}\frac{i^{k-1}\beta_k}{k!}\frac{\partial^k A}{\partial T^k}=i\gamma\left(\left|A\right|^2 A+\frac{i}{\omega_0}\frac{\partial}{\partial T}\left(\left|A\right|^2 A\right)-T_R A\frac{\partial\left|A\right|^2}{\partial T}\right), \tag{1}$$

where A is amplitude of the pulse envelope, α is the fiber loss, β is the dispersion coefficient, γ is the nonlinear coefficient, T_R is Raman coefficient and the term containing ω_0 describes self-steepening effect of fiber. Eq. (1) contains almost all the high-order effect terms, so it should be able to describe the pulse evolution in most single-mode fibers in theory. But Eq. (1) can't be found its analytic solution directly. So, if we want to find the rogue wave solutions, we have to ignore all the high-order terms of Eq. (1) first, and then draw into three dimensionless variables to normalize the equation:

$$u=\frac{A}{\sqrt{P_0}}=\sqrt{\gamma L_D}A,\quad \xi=\frac{z}{L_D},\quad \tau=\frac{T}{T_0}$$

P_0 is the pulse power, T_0 is width of the input pulse and L_D is the dispersion length. After substituting these variables into the equation and ignoring higher-order terms we can get the standard nonlinear Schrödinger equation:

$$i\frac{\partial u}{\partial \xi}+\frac{1}{2}\frac{\partial^2 u}{\partial \tau^2}+|u|^2 u=0, \tag{2}$$

The first-order rogue wave solution of Eq. (2), namely, Peregrine soliton solution (from [7]) is:

$$u_1=\left[-1+\frac{4+8i\xi}{1+4\xi^2+4\tau^2}\right]\exp\left(i\xi\right), \tag{3}$$

And the high-order rogue wave solutions can be solved by modified Darboux transformation which is mentioned in [8]:

$$u_n=\left[(-1)^n+\frac{G_n+iH_n}{D_n}\right]\exp\left(i\xi\right), \tag{4}$$

The parameters G_n, H_n, D_n in the formula are rational polynomials with ξ and τ as variables. Rogue wave of different orders corresponds to different G_n, H_n, D_n. And the higher order of the rogue wave, the more complex the form of these rational polynomials. Darboux transformation is actually an iterative transformation of the equation's solution. As long as we find the corresponding relation between the solution of the equation and some other functions, then we can construct other high-order solutions by iteratively translating the solutions with other functions. The traditional Darboux transformation is one of the common methods to solve integral differential equations. It can be used to construct the first-order soliton solution of the nonlinear equation, the first-order breather solution and the first-order rogue wave solution. However, it can't be used to construct higher-order solutions, because the classical Darboux transformation can't be iterated in the same spectrum. So, if we want to construct higher-order solutions, we need to use the modified Darboux transformation.

3. Simulation results

Now we conduct a simulation about the spatio-temporal locality of rogue wave in an optical fiber, which use numerical method called split-step Fourier method. We set up the nonlinear coefficient γ = 10 $W^{-1}km^{-1}$, the second-order dispersion coefficient β_2 = -1 ps^2/km, and ignore high-order dispersion temporarily. The reason why we make the second-order dispersion coefficient negative is that rogue wave is the result of modulation instability, which only occur in anomalous dispersion regime. Then we set up the width of input pulse is T_0 = 1 ps, the background intensity equals to 1, and the form of the initial pulse is Eq. (3) & Eq. (4) with ξ = -5.

Firstly, let us discuss when the rogue wave will keep its spatio-temporal locality. We find that the rogue wave in fiber keep its spatio-temporal locality only when $|\beta_2|/\gamma P_0 T_0{}^2 = 1$. This relation is not a new formula, it is very common in the field of optical soliton, and article [18] has a discussion of it. The relation means the formation condition of optical soliton with temporal locality in an optical fiber, that is to say, the dispersive interaction and the nonlinear interaction will keep balance in such a condition. And the rogue wave has to meet this condition to keep spatio-temporal locality, reflects the spatio-temporal locality of rogue wave and the temporal locality of optical soliton meet the same principle, that is, they both are the result of the balance of dispersion and nonlinear effect in fiber. In order to satisfy this relation, we set up