Optik 164 (2018) 575–589

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Short note

Quantum breathers associated with modulational instability in 1D ultracold boson in optical lattices involving next-nearest neighbor interactions

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ARTICLE INFO

Article history:
Received 17 January 2018
Received in revised form 17 March 2018
Accepted 18 March 2018

Keywords:
Quantum breathers
Ultracold boson in optical lattices
Modulational instability
Next-nearest neighbor interactions

ABSTRACT

The dynamics and modulation instability of an ultracold gas of bosonic atoms in an optical lattice can be portrayed by a Bose–Hubbard model and the system parameters are mastered by laser light. Based on the time dependent Hartree approximation combined with the semi-discrete multiple-scale method, the equation of motion for single-boson wave function is found analytically, the existence conditions of appearance of bright stationary localized solitons solutions of this quantum Bose–Hubbard model are discussed. We find that the introduction of the next nearest neighbor interactions (NNNI) may change the stability property of the plane waves and may predict the formation of modulational instability in the wave number $k = k_{max}$ and $k = k_{eBZ}$ in the system. With the help of stationary localized single-boson wave functions obtained, the quantized energy level and the quantum breather state are determined. The performance of the analytical results are checked by numerical calculations. Furthermore, we have shown that the presence of the NNNI affect significatively the shape of the region of modulational instability and it is responsible of the appearance of new region of modulational instability that occurs for the $k = k_{max}$ carrier wave. The formation conditions of the modulational instability region predicted by the analytical analysis of stationary localized solutions in the wave number $k = k_{max}$ and $k = k_{eBZ}$ are in good agreement with the forecast respectively.

1. Introduction

During the last few years, ultracold atomic gases have attracted tremendous attention. This is due to the experimental achievement of quantum degeneracy for bosonic [1] and fermionic [2] gases. Atoms in optical lattice are considered as

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https://doi.org/10.1016/j.ijleo.2018.03.059

one of more promising systems for which special emphasis has opened a new window to simulate many-body quantum systems. The Bose–Hubbard (bosonic gase) and Hubbard (fermion gase) are two most physical models introduced that can be simulated in optical lattice. Recently, the experimental developments in the manipulation of ultracold atoms have opened a new fascinating research field, for instance, the analysis of strongly-correlated atomic gases. In this way, experimental results on Bose–Einstein condensates in optical lattices [3] and experiments on ultracold gases in optical lattices, such as, the observation of the Mott-insulator to superfluid transition have already been reported [4]. The Bose–Hubbard model which can be used to describe the dynamics of an ultracold gas of bosonic atoms [7], has attracted large interest and can be considered today as a promising candidate for their practical implications in quantum computing because quantum computation is very sensitive to even smallest disturbances from environment [6,5]. In the classical limit, it is equivalent to a large number of bosons and can be well approximated by the Discrete Nonlinear Schrodinger (DNS) which is one of the most studied nonlinear lattice systems [8]. On experimental development in optical lattices, ultracold atoms offers the unprecedented potential to study the nonlinear properties of many-body interacting [9].

In classical nonlinear lattices, discrete breathers which are localized nonlinear excitation, generic time periodic and spatially localized solutions of the nonlinear systems [10,11], can play important role in both energy storage and transport [12] and have been observed experimentally and theoretically in many physical systems [13–18]. MacKay and Aubry were the first to establish the existence of breathers excitations in 1D nonlinear lattices [19].

However, their quantum counterparts called quantum breather, according to Fleurov [20], can be defined as superpositions of nearly degenerate many quanta bound states characterized with long times to tunnel from one lattice site to another. Quantum breather is nowadays, one of the most interesting domain that attracts the attention of searchers in many physical systems. In the Bose–Hubbard model, few studies have been devoted to prove the existence of quantum breathers. For instance, Tang [21] investigated the existence of quantum two-breathers formed by ultracold bosonic atoms in optical lattices for a large number of bosons. More recently, we probed the quantum signature of breathers in 1D ultracold bosons in optical lattices involving next-nearest neighbor interactions for a few number of bosons [22] and the importance of nearest and next-nearest-neighbor off-site interactions has also been emphasized experimentally in the extended Bose–Hubbard model [23]. Quantum breathers states which are the solitons bright solution are directly linked to the modulational instability of plane waves as demonstrated by Lai and Siever [24] that, the modulational instability is the basic mechanism for the formation of nonlinear localized structures in spin systems.

The modulational instability is a basic process in the theory of nonlinear wave [25–29]. Modulational instability is characterized by an exponential growth of amplitude perturbations occurring under the interplay between nonlinearity and dispersion during propagation. The best way to predict the formation of intrinsic localized modes in nonlinear lattice is Modulational instability [30–32]. The generation of localized states in nonlinear lattices by the modulational instability was established firstly by Kivshar and Peyrard [33]. Kivshar showed that, the formation of bright intrinsic localized mode is instantly linked to the modulational instability of carrier waves [30].

However, in the case of the Bose–Hubbard model, few studies have been devoted just to analyze the non-equilibrium quantum dynamics of a Bose-Einstein condensate and to study the density modulations associated with the dynamical instability [34,35]. From the foregoing, it is clear that the study of the modulation instability in 1D ultracold boson in optical lattices is not completely achieved and needs more attention especially when the NNNI are included in the system.

We will in this paper study the dynamics of ultracold bosonic atoms loaded in an optical lattice involving the NNNI and will look for the existence condition of quantum breathers associated with modulational instability in the system.

The organization of this paper is as follows: In the next section, we rewrite the Hamiltonian of an ultracold gas of bosonic atoms in an optical lattice including the NNNI describes by the Bose–Hubbard model. In section III, we use the time dependent Hartree approximation to derive the equation of motion describing the dynamics of ultracold gas of bosonic atoms in an optical lattice. From the dispersion curves, the existence regions of bright and dark solitons stationary localized solutions are discussed in the half of the Brillouin zone. In section VI, a numerical simulation is used to check our analytical prediction. In section V, the modulational instability conditions are studied through the linear stability analysis and numerical verifications of analytical forecast are performed. The last section is the conclusion.

2. The model Hamiltonian involving the NNNI

Since Jaksch et al. [7] have shown that the dynamics of the bosonic atoms on the optical lattices realize a Bose-Hubbard model, describing the hopping of bosonic atoms between the lowest vibrational states of the optical lattice sites for which the system can be controlled by the laser parameters and configurations [36]. It is important to use the Bose–Hubbard model to describe the dynamics of an ultracold gas of bosonic atoms because this model provides good description of utracold bosonic atoms in the limit of sufficiently deep optical lattice. An optical lattice is formed by pairs of counter-propagating laser beams, which creates effective potential that traps ultracold atoms. An ultracold atom loaded into a three-dimensional (3D) optical