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Dark, bright optical and other solitons with conformable space-time fractional second-order spatiotemporal dispersion

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ABSTRACT

This paper constructs dark, bright, combined dark-bright, singular, combined singular soliton optical and singular periodic solutions to the conformable space-time fractional second order nonlinear Schrödinger equation with group velocity dispersion coefficient and spatiotemporal dispersion by using the extended sinh-Gordon equation expansion method. The constraint conditions for the existence of valid soliton solutions are given. The reported results in this study may be helpful in explaining the physical meaning of the studied model.

1. Introduction

Nonlinear Schrödinger equations (NLSEs) are prototypical dispersive nonlinear evolution equations arising in different fields of nonlinear science, such as; optical fibers, hydrodynamics, condense matter physics, plasma physics and so on. These kind of equations describe the construction of some optoelectronic devices, electromagnetic wave propagation, pulse propagation along orthogonal polarization axes in nonlinear optical fibers and in wavelength-division-multiplexed systems [1–11]. The theory of optical solitons is becoming one of the interesting topics of study. Solitons are the stable localized waves that propagate in a nonlinear medium without amplitude attenuation and shape change due to the balance between the dispersion and nonlinearity [12–14].

For the past two decades, considerable number of studies on various kind of NLSEs have been submitted to literature. It is known that these equations describe the dynamics of soliton propagation through optical fibers for long distances, several results have been reported, some of these models are complex Ginzburg-Landau equation, the cubic-quartic nonlinear Schrödinger's equation, perturbed nonlinear Schrödinger's equation with anti-cubic nonlinearity, the Gerdjikov-Ivanov model, the Biswas-Milovic equation [15–21], and several others [22–38]. This study is aimed at using the extended sinh-Gordon equation expansion method [39–41] to construct various soliton solutions to the conformable space-time fractional

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second order NLSE with group velocity dispersion (GVD) coefficient and spatiotemporal dispersion(STD). The governing model is given by

$$i(\delta D_t^{\alpha}\psi + D_x^{\beta}\psi) + \nu D_t^{2\alpha}\psi + \gamma D_x^{2\beta}\psi + |\psi|^2\psi, \quad \alpha, \beta \in (0, 1), \tag{1.1}$$

where ψ is a function of the space coordinate along which the wave is travelling; x and the time; t which stands for the complex-valued wave profile. The parameter δ is proportional to a ratio of group speeds, $\nu = \frac{s}{2}$ and γ are the coefficients of group velocity dispersion and spatial dispersion, respectively [42,43]. When $\alpha = 1$, Eq. (1.1) becomes the original NLSE with group velocity dispersion coefficient and second-order spatiotemporal dispersion coefficients [42,43].

Fractional calculus is a very important and useful branch of mathematics having a wider range of applications in the various fields of nonlinear science [44]. There are different definitions of fractional differential equations available in the literature, this include; the Riemann-Liouville, Caputo and Grunwald-Letnikov definitions, Atangana-Baleanu derivative in Caputo sense, Atangana-Baleanu fractional derivative in Riemann-Liouville sense [45,46]. Recently, a new simple definition of the fractional derivative known as conformable fractional derivative has been introduced by Khalil et al. [47]. Various studies in this context have been submitted to the literature [48–58].

2. The conformable fractional derivative

In this section, some basic definition, properties and theorem about conformable fractional derivative are discussed [47,59].

Definition 1. Let $g : (0, \infty) \to \mathbb{R}$, then the conformable fraction derivative of g of order α is defined as

$$T_{\alpha}(g)(t) = \lim_{\epsilon \to 0} \frac{g(t + \epsilon t^{1-\alpha}) - g(t)}{\epsilon}, \quad t > 0, \quad 0 < \alpha < 1. \tag{2.1}$$

Here, some basic properties of conformable fractional derivative [47,59,60] are presented.

1. $T_{\alpha}(bg + ch) = bT_{\alpha}(g) + cT_{\alpha}(h)$, $b, c \in \mathbb{R}$,
2. $T_{\alpha}(t^{\lambda}) = \lambda t^{\lambda-\alpha}$, $\lambda \in \mathbb{R}$,
3. $T_{\alpha}(gh) = gT_{\alpha}(h) + hT_{\alpha}(g)$,
4. $T_{\alpha}\left(\frac{g}{h}\right) = \frac{hT_{\alpha}(g) - gT_{\alpha}(h)}{h^2}$,
5. if g is differentiable, then $T_{\alpha}(g)(t) = t^{1-\alpha}\frac{dg}{dt}$.

Theorem 1. *Let $g, h : (0, \infty) \to \mathbb{R}$ be differentiable and also α differentiable functions, then the following rule holds:*

$$T_{\alpha}(g \circ h)(t) = t^{1-\alpha} h\prime(t) g\prime(h(t)). \tag{2.2}$$

3. The extended ShGEEM

In this sections, the analysis of the sinh-Gordon equation expansion method is presented.
To use the ShGEEM, the following steps need to be followed:
Step-1: Consider the following space-time fractional nonlinear differential equation:

$$P(D_x^{\beta}\psi, \psi^2 D_x^{2\beta}\psi, D_t^{\alpha}\psi, D_t^{\alpha}(D_x^{\beta}\psi), \ldots) = 0, \quad \alpha, \beta \in (0, 1), \tag{3.1}$$

where P is a polynomial in ψ, D_x^{β} and D_t^{α} indicate the fractional derivatives of ψ with respect to x and t, respectively.
Consider the following space-time fractional travelling wave transformation:

$$\psi = \Psi(\xi), \quad \xi = \frac{x^{\beta}}{\beta} + \upsilon \frac{t^{\alpha}}{\alpha}. \tag{3.2}$$

Putting Eq. (3.2) into Eq. (3.1), yields the following nonlinear ordinary differential equation (NODE):

$$Q(\Psi, \Psi\prime, \Psi'', \Psi^2\Psi\prime, \ldots) = 0, \tag{3.3}$$

where Q is a polynomial in Ψ and the superscripts indicate the ordinary derivative of Ψ with respect to ξ.
Step-2: Solution of Eq. (3.3) is assumed to posses the following form [39]:

$$\Psi(\theta) = \sum_{j=1}^{m} [b_j \sinh(\theta) + a_j \cosh(\theta)]^j + a_0, \tag{3.4}$$

where a_0, a_j, b_j ($j = 1, 2, \ldots, m$) are constants to be determine later and θ is a function of ξ which satisfies the following NODEs:

$$\theta\prime = \sinh(\theta), \tag{3.5}$$