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Optical solitons to the resonant nonlinear Schrödinger equation with both spatio-temporal and inter-modal dispersions under Kerr law nonlinearity

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ABSTRACT

This study uses the extended sinh-Gordon equation expansion method in constructing various optical soliton solutions to the resonant nonlinear Schrödinger equation with both spatio-temporal and inter-modal dispersions. Resonant nonlinear Schrödinger equation expresses the propagation dynamics of optical solitons and Madelung fluids. Dark, bright, combined dark–bright and singular optical solitons are successfully constructed. Under the choice of suitable values of parameters, the two-dimensional, three-dimensional and the contour graphs to some of the acquired results are plotted. The reported results may be useful in explaining the physical meaning of the studied model.

1. Introduction

Many complex nonlinear aspects arising in various fields of nonlinear sciences, such as; optical fibers, fluid dynamics, plasmas physics, quantum mechanics etc can be expressed in the form of nonlinear Schrödinger equations (NLSEs) [1–5]. The theory of optical solitons is one of the interesting topics for the investigation of soliton propagation through nonlinear optical fibers [6]. Optical solitons are restrained electromagnetic waves that stretch in nonlinear dispersive media and allow the intensity to remain unchanged due to the balance between dispersion and nonlinearity effects [7]. Various computational approaches have been used to address different kind of NLSEs, such as the generalized Kudryashov method [8], the F-expansion method [9], the modified $\exp(-\varphi(\zeta))$-expansion function method [10], the (G'/G)-expansion method [11], the modified simple equation method [12], the improved $\tan(F(\Phi(\xi))/2)$-expansion method [13], the sine-Gordon expansion method [14], the ansatz approach [15,16], the extended trial equation method [17], the improved adomian decomposition method [18], the method of dynamical systems [19], the Bäcklund transformations [20], the method of undetermined coefficients [21], the semi-inverse variational principle [22,23], the trial solution method [24,25], and many other powerful integral schemes [26–39].

This study focuses on construction of various optical soliton solutions to the resonant nonlinear Schrödinger equation (R-NLSE) with both spatio-temporal (STD) and inter-modal dispersions (IMD) [40] by using the powerful extended sinh-Gordon equation expansion method (ShGEEM) [41–43].

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The resonant nonlinear Schrödinger equation [40,44] is given by

$$i(\psi_t - \delta\psi_x) + \alpha\psi_{xx} + \beta\psi_{xt} + \lambda F(|\psi|^2)\psi + \gamma\left(\frac{|\psi|_{xx}}{|\psi|}\right)\psi = 0, \tag{1.1}$$

where $\psi(x, t)$ is the complex wave profile, x and t are the spatial and temporal variables, respectively. In Eq. (1.1), α, β and δ stand for the coefficients of group-velocity dispersion, STD and IMD, respectively, while λ and γ stand for the coefficients of non-Kerr law nonlinearity and resonant nonlinearity, respectively [40,44].

The R-NLSE is used to models the nonlinear dynamics of optical solitons and Madelung fluids [40,45]. Various integral schemes have been adopted to investigate different kinds of R-NLSE [46–53].

2. The extended ShGEEM

This sections discuses the analysis of the extended sinh-Gordon equation expansion method.

To apply the ShGEEM, we go by the following steps:

Step-1: Consider the following NPDE:

$$F(\psi_x, \psi^2\psi_{xx}, \psi_t, \psi_{xt}, \ldots) = 0, \tag{2.1}$$

where F is a polynomial in ψ, the subscripts indicate the partial derivative of ψ with respect to x or t.

Substituting the traveling wave transformation

$$\psi = \Phi(\eta), \quad \eta = x - ct \tag{2.2}$$

into Eq. (2.1), yields the following NODE:

$$Q(\Phi, \Phi', \Phi'', \Phi^2\Phi', \ldots) = 0, \tag{2.3}$$

where Q is a polynomial in Φ and the superscripts indicate the ordinary derivative of Φ with respect to η.

Step-2: The trial solution to Eq. (2.3) is assumed to be of the form [41]

$$\Phi(\theta) = \sum_{j=1}^{k}[b_j \sinh(\theta) + a_j \cosh(\theta)]^j + a_0, \tag{2.4}$$

where a_0, a_j, b_j (j = 1, 2, ..., k) are constants to be determine later and θ is a function of η that satisfies the following ordinary differential equation:

$$\theta' = \sinh(\theta). \tag{2.5}$$

To find the value of k, the homogeneous balance principle is applied on the highest derivatives and highest power nonlinear term in Eq. (2.3).

Eq. (2.5) is obtained from sinh-Gordon equation [41] given as

$$\psi_{xt} = \lambda \sinh(q). \tag{2.6}$$

Eq. (2.5) posses the following solutions [41]:

$$\sinh(\theta) = \pm csch(\eta) \quad \text{or} \quad \sinh(\theta) = \pm i\, sech(\eta) \tag{2.7}$$

and

$$\cosh(\theta) = \pm\coth(\eta) \quad \text{or} \quad \cosh(\theta) = \pm\tanh(\eta), \tag{2.8}$$

where $i = \sqrt{-1}$.

Step-3: Inserting Eq. (2.4), its derivatives under fixed value of k along with Eq. (2.5) into Eq. (2.3), yields a polynomial equation in $\theta'^l \sinh^i(\theta)\cosh^j(\theta)$ (l = 0, 1 and i, j = 0, 1, 2, ...). We collect a group of over-determined nonlinear algebraic equations in a_0, a_j, a_j, c by setting the coefficients of $\theta'^l \sinh^i(\theta)\cosh^j(\theta)$ to zero.

Step-4: The secured set of over-determined nonlinear algebraic equations is then solved with aid of symbolic software to determine the values of the parameters a_0, a_j, b_j, c.

Step-5: Based on Eqs. (2.7) and (2.8), Eq. (2.1) posses the following forms of solutions:

$$\Phi(\eta) = \sum_{j=1}^{k}[\pm ib_j\, sech(\eta) \pm a_j \tanh(\eta)]^j + a_0, \tag{2.9}$$

$$\Phi(\eta) = \sum_{j=1}^{k}[\pm b_j\, csch(\eta) \pm a_j \coth(\eta)]^j + a_0. \tag{2.10}$$