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Optical soliton perturbation for Radhakrishnan–Kundu–Lakshmanan equation with a couple of integration schemes

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ABSTRACT

This paper obtains optical soliton solutions to the perturbed Radhakrishnan–Kundu–Lakshmanan equation by trial equation method and modified simple equation algorithm. There are two types of nonlinear fibers studied in this paper. They are Kerr law and power law. Bright, dark and singular soliton solutions are derived. Additional solutions such as singular periodic solutions also fall out of the integration schemes.

1. Introduction

The dynamics of soliton propagation through any form of optical waveguide is modeled by a variety of nonlinear evolution equations (NLEEs). A few of them are nonlinear Schrödinger's equation, Schrödinger–Hirota equation, Manakov model, Sasa–Satsuma equation, Chen–Lee–Liu equation, Lakshmanan–Porsezian–Daniel model and many others. This paper will address one such NLEE. This is the Radhakrishnan–Kundu–Lakshmanan (RKL) equation along with a couple of perturbation terms that will not destroy its integrability. There are several integration tools in the literature that has been applied to study RKL equation and other NLEEs to retrieve solitons and other solutions [1–10]. This paper will implement a couple of similar

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powerful integration schemes to secure soliton solutions to the perturbed RKL equation that is considered in Kerr law and power law media. These are trial equation method and modified simple equation scheme. They will successfully retrieve bright, dark and singular soliton solutions to the model, subjected to certain parameter restrictions that are all enumerated in the next couple of sections.

2. Review of trial equation method

In this section, the main steps of the algorithm are succinctly enumerated [2,3,8]:

Step-1: Suppose we have a nonlinear evolution equation (NLEE) with time-dependent coefficients

$$P(u, u_t, u_x, u_{tt}, u_{xt}, u_{xx}, \ldots) = 0 \tag{1}$$

which can be transformed to an ordinary differential equation (ODE)

$$Q\left(U, U', U'', U''', \ldots\right) = 0 \tag{2}$$

with a travelling wave hypothesis $u(x, t) = U(\xi)$, $\xi = x - vt$, where $U = U(\xi)$ is the unknown function, Q is a polynomial with variable U and its derivatives. If all of the terms contain derivatives, then Eq. (2) can be integrated with the integration constants taken to be zeros, without any loss of generality.

Step-2: Choose the trial equation

$$\left(U'\right)^2 = F(U) = \sum_{l=0}^{N} a_l U^l \tag{3}$$

where a_l ($l = 0, a, \ldots, N$) are constants to be determined. Substituting Eq. (3) and other derivative terms such as U'' or U''' and so on into Eq. (2) yields a polynomial $G(U)$ in U. The value of N can be obtained from the balancing principle. Next, setting the coefficients of $G(U)$ to zero, we obtain a system of algebraic equations. Solving this system, we can determine v and the values of $a_0, a_1, \ldots, a_N$.

Step-3: Recast Eq. (3) in the integral form

$$\pm\left(\xi - \xi_0\right) = \int \frac{dU}{\sqrt{F(U)}} \tag{4}$$

According to the discriminants of the integrand, we can classify the roots of $F(U)$, and solve for the integral in Eq. (4). Thus, we recover the exact solution of Eq. (1).

2.1. Kerr law

The dimensionless form of perturbed RKL equation with Kerr law is given by [4–6,9,10]

$$iq_t + aq_{xx} + b|q|^2 q = i\alpha q_x + i\lambda\left(|q|^2 q\right)_x + i\mu\left(|q|^2\right)_x q - i\gamma q_{xxx}. \tag{5}$$

In (5), the dependent variable $q(x, t)$ represents the complex valued wave function with the independent variables being x and t that represents space and time respectively. The first term on the left side represents the temporal evolution of the nonlinear wave, while the coefficient a is the group-velocity dispersion (GVD) and b represents the coefficient of nonlinearity. On the right hand side of (5), α is the inter-modal dispersion, λ represents the coefficient of self-steepening for short pulses, μ is the higher-order dispersion coefficient and γ represents the third order dispersion term.

In order to solve Eq. (5) by the trial equation method, we use the following wave transformation

$$q(x, t) = U(\xi)e^{i\phi(x,t)} \tag{6}$$

where $U(\xi)$ represents the shape of the pulse, $\xi = x - vt$ and $\phi(x, t) = -\kappa x + \omega t + \theta$. The function $\phi(x, t)$ is the phase component of the soliton, κ is the soliton frequency, while ω is the wave number, θ is the phase constant and v is the velocity of the soliton.

Substituting Eq. (6) into Eq. (5) and then decomposing into real and imaginary parts yields a pair of relations. The real part gives

$$(a + 3\gamma\kappa)U'' - \left(\omega + a\kappa^2 + \gamma\kappa^3 + \alpha\kappa\right)U + (b - \kappa\lambda)U^3 = 0 \tag{7}$$

while the imaginary part gives

$$\gamma U'' - \left(\alpha + v + 2a\kappa + 3\gamma\kappa^2\right)U - \frac{3\lambda + 2\mu}{3}U^3 = 0. \tag{8}$$

As the same function $U(\xi)$ satisfies both Eqs. (7) and (8), we obtain the following constraint conditions

$$\frac{a + 3\gamma\kappa}{\gamma} = \frac{\omega + a\kappa^2 + \gamma\kappa^3 + \alpha\kappa}{\alpha + v + 2a\kappa + 3\gamma\kappa^2} = -\frac{3(b - \kappa\lambda)}{3\lambda + 2\mu} \tag{9}$$