Optik 163 (2018) 137–141

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Original research article

Optical soliton perturbation with Radhakrishnan–Kundu–Lakshmanan equation by Lie group analysis

Anupma Bansal [a], Anjan Biswas [b,c,d], Mohammad F. Mahmood [e], Qin Zhou [f,*], Mohammad Mirzazadeh [g], Ali Saleh Alshomrani [c], Seithuti P. Moshokoa [d], Milivoj Belic [h]

[a] *Department of Mathematics D. A. V. College for Women, Ferozepur 152001, Punjab, India*
[b] *Department of Physics, Chemistry and Mathematics, Alabama A&M University, Normal, AL 35762, USA*
[c] *Department of Mathematics, Faculty of Science, King Abdulaziz University, Jeddah 21589, Saudi Arabia*
[d] *Department of Mathematics and Statistics, Tshwane University of Technology, Pretoria 0008, South Africa*
[e] *Department of Mathematics, Howard University, Washington, DC 20059, USA*
[f] *School of Electronics and Information Engineering, Wuhan Donghu University, Wuhan 430212, People's Republic of China*
[g] *Department of Engineering Sciences, Faculty of Technology and Engineering, East of Guilan, University of Guilan, P.C. 44891-63157 Rudsar-Vajargah, Iran*
[h] *Science Program, Texas A&M University at Qatar, PO Box 23874, Doha, Qatar*

A R T I C L E I N F O

Article history:
Received 26 January 2018
Accepted 27 February 2018

OCIS:
060.2310
060.4510
060.5530
190.3270
190.4370

Keywords:
Solitons
Parabolic law nonlinearity
Weak nonlocal nonlinearity

A B S T R A C T

This paper conducts Lie group analysis of the perturbed Radhakrishnan–Kundu– Lakshmanan equation to retrieve optical soliton solutions. This powerful integration scheme retrieves bright and dark optical soliton solutions to the model that is studied with power law nonlinearity. The existence criteria of such solitons are also presented.

1. Introduction

The study of optical solitons and other nonlinear evolution equations is one of the active areas of research in mathematical physics, applied mathematics and telecommunication sciences. There are several mathematical approaches that can be applied to these studies [1–10]. One of the models that studies dispersive optical solitons is the Radhakrishnan–Kundu–Lakshmanan (RKL) equation. This is the familiar nonlinear Schrödinger's equation that is considered with a couple of perturbations that are due to third order dispersion (3OD) and the self-steepening effect. This model has been studied since many years by a variety of methods that led to the retrieval of bright and dark optical solitons [2,3,10].

* Corresponding author.
E-mail address: qinzhou@whu.edu.cn (Q. Zhou).

https://doi.org/10.1016/j.ijleo.2018.02.104

These are the exp-function scheme and the method of undetermined coefficients. This paper will apply one of the most powerful mathematical tools to address the RKL equation that appears with power law nonlinearity and nonlinear dispersion. The self–steepening effect and nonlinear dispersion terms are considered with full nonlinearity. Lie symmetry analysis will retrieve bright and dark singular soliton solutions to this model. The details are discussed in the rest of the paper.

1.1. Governing model

The dimensionless form of the generalized RKL equation with power law nonlinearity, in presence of nonlinear dispersion is given by [2,3,10]

$$iq_t + aq_{xx} + b|q|^{2m}q = i\left[\lambda\left(|q|^{2m}q\right)_x - \gamma q_{xxx} + \nu\left(|q|^{2m}\right)_x q\right], \tag{1}$$

where the first term represents the evolution term, while the second and third terms represent the group velocity dispersion and nonlinear terms respectively. In the right hand side, the terms due to λ, γ, ν are known as self-steeping term, 3OD and nonlinear dispersion, respectively, with constant coefficients. The parameter m indicates the case of power law nonlinearity which is usually the general form of Kerr law nonlinearity. It also needs to be noted that for the stability of the soliton solutions of (1), it is necessary to have $0 < m < 2$. The special case for $\nu = 0$ is called the RKL equation.

2. Lie group analysis

In this section, we will perform Lie symmetry analysis [1,4–9] for generalized RKL Eq. (1). For this, firstly let us assume

$$q(x,t) = u(x,t) + iv(x,t). \tag{2}$$

On substituting (2) in (1) and then, separating real and imaginary parts we get

$$-v_t + au_{xx} + bu(u^2+v^2)^m = -m(\lambda+\nu)(u^2+v^2)^{m-1}(2uvu_x + 2v^2v_x) - \lambda(u^2+v^2)^m v_x + \gamma v_{xxx}, \tag{3}$$

and

$$u_t + av_{xx} + bv(u^2+v^2)^m = m(\lambda+\nu)(u^2+v^2)^{m-1}(2u^2u_x + 2uvv_x) + \lambda(u^2+v^2)^m u_x - \gamma u_{xxx}. \tag{4}$$

Let us consider the Eqs. (3) and (4) are invariant under following transformations:

$$\begin{bmatrix} u \\ v \\ x \\ t \end{bmatrix} \longrightarrow \begin{bmatrix} u \\ v \\ x \\ t \end{bmatrix} + \epsilon \begin{bmatrix} \eta(x,t,u) \\ \phi(x,t,u) \\ \xi(x,t,u) \\ \tau(x,t,u) \end{bmatrix}, \tag{5}$$

where ϵ is an infinitesimal parameter. On invoking the invariance criterion, the following relation from the coefficients of the first order of ϵ in (3) and (4) are deduced as:

$$\begin{aligned} &-\phi^t + a\eta^{xx} + 2mbu(u^2+v^2)^{m-1}(u\eta + v\phi) + b\eta(u^2+v^2)^m = -\lambda((u^2+v^2)^m\phi^x) \\ &-\lambda(2mv_x(u^2+v^2)^{m-1}(u\eta+v\phi)) - m(\lambda+\nu)((u^2+v^2)^{m-1}(2uv\eta^x + 2uu_x\phi + 2u_xv\eta + 2v^2\phi^x + 4vv_x\phi)) \\ &-(\lambda+\nu)2m(m-1)(u^2+v^2)^{m-2}(u\eta+v\phi)(2uvu_x + 2v^2v_x)) + \gamma\phi^{xxx}, \end{aligned} \tag{6}$$

and

$$\begin{aligned} &\eta^t + a\phi^{xx} + 2mbv(u^2+v^2)^{m-1}(u\eta+v\phi) + b\phi(u^2+v^2)^m = \lambda((u^2+v^2)^m\eta^x) \\ &+2m\lambda u_x(u^2+v^2)^{m-1}(u\eta+v\phi) + m(\lambda+\nu)(u^2+v^2)^{m-1}(2u^2\eta^x + 4uu_x\eta + 2uv\phi^x + 2u\phi v_x + 2vv_x\eta) \\ &+2m(m-1)(\lambda+\nu)(u^2+v^2)^{m-2}(u\eta+v\phi)(2u^2u_x + 2uvv_x) - \gamma\eta^{xxx}, \end{aligned} \tag{7}$$

where η^t, ϕ^t, η^x, ϕ^x, η^{xx}, ϕ^{xx}, η^{xxx}, ϕ^{xxx} are extended (prolonged) infinitesimals acting on an enlarged space that includes all derivatives of the dependent variables u_t, v_t, u_x, v_x, u_{xx}, v_{xx}, u_{xxx}, v_{xxx}. The infinitesimals are determined from invariance