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Diffraction of waves by a wedge residing between two different media

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ABSTRACT

The scattering problem of plane waves by a perfectly electric conducting wedge, residing at the planar interface of two media with different electromagnetic properties, is investigated. The structure of the scattered geometrical optics waves is used in order to determine the diffracted waves, which are expressed in terms of a split function that arises in the solution of the diffraction problem by a resistive half-plane. The uniform diffraction fields are obtained with the aid of uniform theory of diffraction. The behavior of the total, total geometrical optics and diffracted waves are studied numerically.

1. Introduction

The diffraction phenomenon of waves by a wedge is a canonical problem that finds many areas of applications in optics. The high-frequency expressions of the diffracted fields by soft and hard wedges underline the foundations of the geometrical theory of diffraction [1] and uniform theory of diffraction (UTD) [2]. However, a more fundamental problem, related with wedge, has not been investigated in the literature yet, to our knowledge. This problem describes a wedge, residing between two media with different permittivitiy and permeability. In fact the scattering of waves by a wedge in a single medium is a special case of this scenario. Similar scattering problems were studied for a perfectly conducting [3–6] and resistive [7] half-planes. However, the diffraction field expressions, obtained by these studies contain spurs. We put forth alternative solutions, which do not have any spurious fields, for the perfectly conducting half-screen problem [8–10], but these expressions are approximate.

The aim of this paper is to investigate the diffraction problem of plane waves by a soft wedge, located on a planar junction between two different media. First of all the initial fields, which occur on a planar whole interface between two different media, will be obtained. Then the total geometric optics (GO) waves will be determined for the problem under consideration. The scattered GO fields will be evaluated by subtracting the initial fields from the total GO waves. The diffracted fields, excited by the wedge, will be obtained with the aid of a relation, put forth by us [11–13]. The uniform diffraction waves will be derived by using UTD [2,14]. The behaviors of the total, total GO and diffracted waves will be analyzed numerically.

A time factor of $\exp(j\omega t)$ is assumed and suppressed throughout the paper. ω is the angular frequency.

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2. Introduction of the method

We suppose an incident wave that has the time harmonic electric field component of u_i interacts with an obstacle on its path of propagation. The wave satisfies the Helmholtz equation

$$\nabla^2 u_i + k^2 u_i = 0 \tag{1}$$

where k is the wave-number. The total field of

$$u_T = u_i + u_S \tag{2}$$

is excited when the incident wave interacts with an obstacle on its path of propagation [15]. u_i is the wave distribution when there is not any scatterer in the medium. u_S is the scattered field and shows the effect of the obstacle on the incident wave. In some problems, like junction diffraction or canonical objects between different media, it is more suitable to redefine Eq. (2) as

$$u_T = u_{in} + u_S \tag{3}$$

for u_{in} represents the initial fields that exist in space when the scattering object is excluded [11–13]. Eq. (3) can be further expressed by

$$u_{TGO} + u_{Td} = u_{in} + u_{SGO} + u_{Sd} \tag{4}$$

in terms of the diffracted and GO waves. Eq. (4) can be decomposed into two parts as

$$u_{TGO} = u_{in} + u_{SGO} \tag{5}$$

and

$$u_{Td} = u_{Sd} \tag{6}$$

since the initial fields do not contain diffracted wave. We will show the diffracted field by u_d from now on. Our previous studies showed a relation between the scattered GO fields and diffracted waves [11–13]. Once the scattered GO field is determined, the diffracted field can be constructed by using suitable functions. Generally, the relation of

$$K_+\left(\alpha,\beta\right) K_+\left(\pi \mp \alpha,\beta\right) = \frac{\sin\alpha \sin\beta}{\sin\alpha + \sin\beta} \tag{7}$$

is encountered in the scattering problems by non-perfectly conducting surfaces. $K_+(\alpha,\beta)$ is the split function that arises in the solution of the resistive half-screen problem [16]. It can be introduced by the equation of

$$K_+\left(\alpha,\beta\right) = \frac{4\sqrt{\sin\beta}\sin\dfrac{\alpha}{2}}{\left[1+\sqrt{2}\cos\dfrac{\left(\pi/2\right)-\alpha+\beta}{2}\right]\left[1+\sqrt{2}\cos\dfrac{\left(3\pi/2\right)-\alpha-\beta}{2}\right]} \times \left\{\frac{\psi_\pi\left(\dfrac{\pi}{2}-\alpha+\beta\right)\psi_\pi\left(\dfrac{3\pi}{2}-\alpha-\beta\right)}{\left[\psi_\pi\left(\dfrac{\pi}{2}\right)\right]^2}\right\}^2 \tag{8}$$

where $\psi_\pi(x)$ is the Maliuzhinets function [17] that has the expression

$$\psi_\pi\left(x\right) = \exp\left[-\frac{1}{8\pi}\int_0^x \frac{\pi\sin v - 2\sqrt{2}\pi\sin\left(v/2\right) + 2v}{\cos v}dv\right]. \tag{9}$$

First of all, the scattered GO wave must be determined from the equation

$$u_{SGO} = u_{SGO} - u_{in} \tag{10}$$

as a second step, the diffracted wave will be constructed by using the relation between the scattered GO and diffracted waves. Now we will derive this relation for the problem of wedge diffraction.

A plane wave of

$$u_i = u_0 e^{jk\rho\cos(\varphi-\varphi_0)} \tag{11}$$

hits a soft wedge (total field is equal to zero on the surface) with an angle of incident of φ_0. The wedge is located in a single medium. u_0 is the complex amplitude. The cylindrical coordinates are given by (ρ,φ,z). The geometry of the problem is shown in Fig. 1. The outer angle of the wedge is equal to η. P is the observation point.