Optik 162 (2018) 35–41

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Original research article

Dispersive optical solitons with Schrödinger–Hirota model by trial equation method

Anjan Biswas[a,b,c], Yakup Yildirim[d], Emrullah Yasar[d], Qin Zhou[e,*], Ali Saleh Alshomrani[b], Seithuti P. Moshokoa[c], Milivoj Belic[f]

[a] Department of Physics, Chemistry and Mathematics, Alabama A&M University, Normal, AL 35762, USA
[b] Department of Mathematics, Faculty of Science, King Abdulaziz University, Jeddah 21589, Saudi Arabia
[c] Department of Mathematics and Statistics, Tshwane University of Technology, Pretoria 0008, South Africa
[d] Department of Mathematics, Faculty of Arts and Sciences, Uludag University, 16059 Bursa, Turkey
[e] School of Electronics and Information Engineering, Wuhan Donghu University, Wuhan 430212, People's Republic of China
[f] Science Program, Texas A&M University at Qatar, PO Box 23874, Doha, Qatar

ARTICLE INFO

Article history:
Received 20 January 2018
Accepted 20 February 2018

OCIS:
060.2310
060.4510
060.5530
190.3270
190.4370

Keywords:
Dispersive solitons
Trial equation method

ABSTRACT

This paper obtains bright, dark and singular dispersive optical soliton solutions to Schrödinger–Hirota equation by the aid of trial equation method. Both Kerr and power laws of nonlinearity are studied. Singular periodic solutions are also obtained as a byproduct of this scheme.

1. Introduction

Dispersive optical solitons are a reality when higher order dispersions are present in addition to the necessary group velocity dispersion (GVD) and spatio-temporal dispersion (STD) that provide the balance between GVD, along with STD and nonlinearity. Thus, the governing nonlinear Schrödinger's equation (NLSE) with third order dispersion is transformed to Schrödinger–Hirota equation (SHE), upto the first order of approximation, by Lie transform. This SHE is the standard model that is studied across the globe to address dispersive optical solitons [1–10]. There are several forms of integration architecture that has been successfully implemented in the past to analyze SHE. They are traveling wave hypothesis, method of undetermined coefficients, extended trial equation method, Darboux transform, Bäcklund transform, G'/G – expansion method and several others. All of these methodologies have successfully retrieved dispersive soliton solution to SHE. This paper will apply yet another scheme that will fetch soliton solutions to the model. This is the trial equation method. After a quick review of the scheme, the details are sketched out in subsequent sections. For a complete analysis, both Kerr law and power law nonlinearity are considered.

* Corresponding author.
E-mail address: qinzhou@whu.edu.cn (Q. Zhou).

https://doi.org/10.1016/j.ijleo.2018.02.058

2. Succinct pen-picture of trial equation method

In this section we outline the main steps of the trial equation method as following:

Step-1: Let the nonlinear evolution equation with time–dependent coefficients to be studied take the form:

$$P(u, u_t, u_x, u_{tt}, u_{xt}, u_{xx}, \ldots) = 0 \tag{1}$$

can be converted to an ordinary differential equation:

$$Q(U, U', U'', U''', \ldots) = 0 \tag{2}$$

with a traveling wave hypothesis $u(x, t) = U(\xi)$, $\xi = x - vt$, where $U = U(\xi)$ is an unknown function, Q is a polynomial in the variable U and its derivatives. If all terms contain derivatives, then Eq. (2) are integrated with integration constants taken to be zeros, without any loss of generality.

Step-2: Choose the trial equation

$$\left(U'\right)^2 = F(U) = \sum_{l=0}^{N} \delta_l U^l \tag{3}$$

where δ_l, $(l = 0, 1, \ldots, N)$ are constants to be determined. Substituting Eq. (3) and other derivative terms such as U'' or U''' and so on into Eq. (2) leads to a polynomial $G(U)$ in U. Based on the balancing principle we can determine the value of N. Setting the coefficients of $G(U)$ to zero, we get a system of algebraic equations. Solving this system, we can determine v and values of $\delta_0, \delta_1, \ldots, \delta_N$.

Step-3: Transform Eq. (3) to the integral form

$$\pm\left(\xi - \xi_0\right) = \int \frac{dU}{\sqrt{F(U)}} \tag{4}$$

Based on classification of the discriminant of the polynomial, we classify the roots of $F(U)$, and solve the integral Eq. (4). This, finally gives the exact solutions to Eq. (1).

3. Application to she

This section will detail the application of trial equation method to SHE. The extraction of soliton solutions will be the target. The study will also reveal additional solutions apart from solitons, as will be seen. The analysis will now be split into two subsections depending on the type of nonlinearity in question.

3.1. Kerr law

The dimensionless form of the perturbed SHE with STD and Kerr law nonlinearity is given by [1–3]

$$iq_t + aq_{xx} + bq_{xt} + c|q|^2 q + i\left(\gamma q_{xxx} + \sigma |q|^2 q_x\right) = i\alpha q_x + i\lambda\left(|q|^2 q\right)_x + i\mu\left(|q|^2\right)_x q \tag{5}$$

where a represents GVD and b is the STD term. From perturbation terms, γ represents the third order dispersion (3OD) coefficient, μ and σ represent the nonlinear dispersion, α is the inter-modal dispersion and λ is the self steepening term. The inclusion of STD terms is a necessity to provide well-posedness to the model, a fact that was first pointed out during 2012 [7,9].

In order to solve this equation by the trial equation method, the following solution structure is selected

$$q(x, t) = U(\xi) e^{i\phi(x,t)} \tag{6}$$

where the wave variable ξ is given by

$$\xi = k(x - vt). \tag{7}$$

Here, $U(\xi)$ represents the amplitude component of the soliton solution and v is the speed of the soliton, while the phase component $\phi(x, t)$ is defined as

$$\phi(x, t) = -\kappa x + \omega t + \theta, \tag{8}$$

where κ is the frequency of the soliton, ω is the wave number, while θ is the phase constant.

Substituting (6) into (5) and decomposing into real and imaginary parts lead to

$$k^2(a - bv + 3\gamma\kappa)U'' - \left(\omega + a\kappa^2 - b\kappa\omega + \gamma\kappa^3 + \alpha\kappa\right)U + (c + \kappa\sigma - \kappa\lambda)U^3 = 0, \tag{9}$$

$$\gamma k^3 U'' + k\left(b\kappa v + b\omega - v - 2a\kappa - 3\gamma\kappa^2 - \alpha\right)U - \frac{k}{3}(3\lambda - \sigma + 2\mu)U^3 = 0. \tag{10}$$