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Optical solitons with Lakshmanan–Porsezian–Daniel model by modified extended direct algebraic method

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ABSTRACT

This paper obtains bright and dark–singular combo solitons for the Lakshmanan–Porsezian–Daniel model by the aid of modified extended direct algebraic method. Both Kerr law and power law nonlinearities are considered. However, it is only the case of Kerr law that led to soliton solutions. Some additional solutions also emerged and they are singular periodic waves and elliptic functions.

1. Introduction

Optical solitons is one of the fastest growing areas of research in the field of telecommunications engineering. In particular, Lakshmanan–Porsezian–Daniel (LPD) model has attracted a lot of attention in the past few years to model soliton transmission through optical fibers and PCF. There are a lot of integration algorithms that have been successfully implemented to extract solitons and solitary waves to a variety of nonlinear evolution equations (NLEEs) [1–20]. Some of these schemes applied to LPD model, in particular, are method of undetermined coefficients [6], semi–inverse variational principle

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[1], extended trial equation method [5], Lie symmetry analysis [4], tanh method [7] and several others. Apart from these schemes, some other algorithms that have been implemented to LPD equation and nonlinear Schrödinger's equation are traveling wave hypothesis [12,13], Jacobi's elliptic function scheme [5] and others. This paper applies the modified extended direct algebraic method to LPD model that is considered with two forms of nonlinear media, namely Kerr law and power law. Thus, bright and dark–singular combo soliton solutions are recovered from this scheme. In addition, singular periodic solutions and elliptic function solutions also fall out of this integration process. The details are enumerated in the rest of the paper.

2. Glimpse of modified extended direct algebraic method

We assume a NLEE for $u(x, t)$ to be in the form

$$P(u, u_t, u_x, u_{tx}, u_{tt}, u_{xx}, \ldots) = 0, \tag{1}$$

where P is a polynomial in its arguments. The essence of the modified extended direct algebraic method can be presented in the following algorithmic steps [9,3,18,17,14,19]:

Step 1: Seeking traveling wave solution to Eq. (2) by taking $u(x, t) = U(\xi)$ and $\xi = kx - \omega t$ Eq. (2) transforms to the ordinary differential equation

$$Q(U, U', U'', \ldots) = 0, \tag{2}$$

where primes denote the derivative with respect to ξ.

Step 2: We introduce the solution $U(\xi)$ of Eq. (3) in the finite series that takes the form [9,3,18,17,14,19]

$$U(\xi) = \sum_{i=-N}^{N} a_i \phi(\xi)^i, \tag{3}$$

where a_i are real-valued constants with $a_N \neq 0$ to be determined, N is a positive integer to be determined. The quantity $\phi(\xi)$ expresses the solution of the following equation [14]

$$\phi'(\xi) = \sqrt{c_0 + c_1\phi(\xi) + c_2\phi^2(\xi) + c_3\phi^3(\xi) + c_4\phi^4(\xi) + c_5\phi^5(\xi) + c_6\phi^6(\xi)}, \tag{4}$$

where c_i are constants and can be discussed as in [20].

Step 3: Determine N. This, usually, can be accomplished by balancing the linear term(s) of highest order with the highest order nonlinear term(s) in Eq. (3).

Step 4: Substituting Eq. (4) together with Eq. (5) into Eq. (3) yields an algebraic equation involving powers of $\phi(\xi)$. Equating the coefficients of each power of $\phi(\xi)$ to zero and discussing the value of c_i [20] gives a system of algebraic equations for a_i. Then, we solve the system with the aid of a computer algebra system (CAS), such as Mathematica or Maple, to determine these constants. On the other hand, depending on the value of parameters c_i [20], the solutions of Eq. (3) are well known to us. Thus, finally, we can obtain exact solutions of the given Eq. (1).

3. Application to LPD model

The dimensionless form of the LPD model with higher order dispersion, full nonlinearity and spatio-temporal dispersion (STD) to be considered in this work is given by [2,8]

$$iq_t + aq_{xx} + bq_{xt} + cF(|q|^2)q = \sigma q_{xxxx} + \alpha(q_x)^2 q^* + \beta |q_x|^2 q + \gamma |q|^2 q_{xx} + \lambda q^2 q^*_{xx} + \delta |q|^4 q, \tag{5}$$

where $q(x, t)$ represents the complex-valued wave profile with x and t are the independent spatial and temporal variables respectively. a is group velocity dispersion, b is the coefficient of quintic nonlinearity, c is the coefficient of nonlinear dispersion, α is the inter-modal dispersion, λ accounts for self-steepening with short pulses, μ is the higher-order dispersion coefficient and m is the full nonlinearity parameter.

Now, using the transformation

$$q(x, t) = P(x, t)e^{i\phi(x,t)}, \quad \phi(x, t) = -\kappa x + \omega t + \theta, \tag{6}$$

where $P(x, t)$ represents the shape of the pulse. Substituting Eq. (6) into Eq. (5) and decomposing into real and imaginary parts yield

$$\begin{aligned} &\sigma P_{xxxx} - (a + 6\sigma\kappa^2)P_{xx} - bP_{xt} - (b\kappa\omega - \omega - a\kappa^2 - \sigma\kappa^4)P - (\alpha + \gamma + \lambda - \beta)\kappa^2 P^3 \\ &\quad + \delta P^5 - cF(P^2)P + (\alpha + \beta)PP_x^2 + (\lambda + \gamma)P^2 P_{xx} = 0, \end{aligned} \tag{7}$$

and

$$(1 - b\kappa)P_t - (2a\kappa + 4\sigma\kappa^3 - b\omega)P_x + 2(\alpha + \gamma - \lambda)\kappa P^2 P_x + 4\sigma\kappa P_{xx} = 0. \tag{8}$$