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Angular position measurement of pulsars based on X-ray intensity correlation

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ABSTRACT

The positioning accuracy of the X-ray pulsar navigation depends on the angular position accuracy of the pulsars. Currently, long baseline interferometer technology provides 0.1 mas pulsar positioning accuracy, which is far away from the requirement of X-ray pulsar navigation. Using classical statistical optics, we studied the relationship between direct observation of the pulsar angular position and second-order correlation. As an application, we give a proposal to realize angular position measurement of pulsars based on X-ray intensity correlation.

1. Introduction

Pulsar is a neutron star with high rotational speed and extremely stable rotation frequency. The long term stability of self rotation frequency of partial millisecond pulsar is better than the most stable cesium atomic clock [1]. As a result, the position coordinates of the pulsars constitute a high precision inertial reference frame. XNAV (X-ray Pulsar Navigation) is a technology demonstration will use photons from X-ray pulsars for navigation and spacecraft attitude determination [2]. However, the pulsar angular position error will lead to poor positioning accuracy of navigation. To reach a 10-meter navigation accuracy, the resolution of pulsar angular position should better than 10μas[3]. At present, the angular positioning accuracy of pulsars is around 0.1 mas by VLBI (Very Long Baseline Interferometry), and it is far away from the requirement of XNAV.

$$\mathrm{D} \cdot \cos\theta = c \cdot \tau \tag{1}$$

The basic principle of VLBI is using multiple radio telescopes to observe the same radio source. Then the angular position of the source can be calculated by the time difference between the arrivals of the radio signal at different telescopes [4]. Actually, this time difference is inversely proportional to the degree of the first-order correlation of the wave field.

The resolution of angular position measurement can be indicated by the Rayleigh Criterion [5],

$$\Delta\theta = 1.22\frac{\lambda}{D} \tag{2}$$

where D is the telescope diameter and λ is the observed wavelength. There are two ways to improve the resolution of telescopes, increasing the telescope caliber or observing shorter wavelengths. Long baseline interference technology makes

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the equivalent caliber of the detection system greatly improved, and its highest resolution is equivalent to a single caliber telescope which diameter is as large as the baseline length.

The X-ray radiated from pulsars ranges from 0.1 nm to 20 nm [6]. Theoretically, if we use the VLBI-like principle to observe the X-ray photons, the resolution of measurement will experience a dramatic increase comparing with the use of S-band radio signal in VLBI.

2. Pulsars angular position and X-ray intensity correlation

However, because of the restriction in detecting technology and optical fabrication in X-ray range, long baseline interferometer technology cannot be directly applied to measure the angular position of X-ray pulsars. Basically, most detectors can only output intensity and the arrival time of X-ray photons [6].

As we known, the first-order correlation of the wave field can be described as the interference,

$$\begin{aligned}
&E(x_1) = Ae^{-i\omega t_1} = A\cos\omega t_1 \\
&E(x_2) = Ae^{-i\omega t_2} = A\cos\omega t_2 \\
&E(x) = E(x_1) + E(x_2) \\
&I(x) = |E(x)|^2 = < E(x)E^*(x) > \\
&= < [E(x_1) + E(x_2)][E^*(x_1) + E^*(x_2)] > \\
&= < E(x_1)E^*(x_1) + E(x_1)E^*(x_2) + E(x_2)E^*(x_1) + E(x_2)E^*(x_2) > \\
&= G^{(1,1)}(x_1,x_1) + G^{(1,1)}(x_1,x_2) + G^{(1,1)}(x_2,x_1) + G^{(1,1)}(x_2,x_2) \\
&= I_0 + 2I_0\cos\omega\tau + I_0 = 2I_0(1+\cos\omega\tau)
\end{aligned} \tag{3}$$

where <...> means the ensemble average, and the intensity distribution $I(x)$is called the fringe pattern. The OPD (Optical Path Difference) x is proportional to the time difference τ between the light from two telescopes. As a result, we can obtain the time difference $\tau$from the amplitude of$I(x)$. Because the first-order serial correlation ($G^{(1,1)}(x_1,x_1)$ or $G^{(1,1)}(x_2,x_2)$) is equal to its initial intensity [7], the first-order cross correlation in charges of the fringe pattern $I(x)$. Therefore, measuring the angular position of pulsars is equivalent to measure the time difference, while measuring the time difference is equivalent to measure the amplitude of the first-order cross-correlation function.

However, the first-order correlation needs frequency and phase information which is easy to be obtained in RF (Radio Frequency) while difficult in X-ray regime [8]. Moreover, it is very difficult to fabricate optical components that function in the X-ray regime, obtain the first-order correlation of pulsar X-ray photons by carrying out a “Young’s interference” type experiment is nearly impossible currently because of the huge focal distance.

The information of photons is erased in the intensity distribution, but can be reproduced in the second-order correlation, which also known as intensity correlation [9,10].

$$\begin{aligned}
&G^{(2,2)}(x_1,x_2) = < E^*(x_1)E^*(x_2)E(x_1)E(x_2) > \\
&= < E^*(x_1)E(x_1) >< E^*(x_2)E(x_2) > + < E^*(x_1)E(x_2) >< E^*(x_2)E(x_1) > \\
&= G^{(1,1)}(x_1,x_1)\cdot G^{(1,1)}(x_2,x_2) + G^{(1,1)}(x_1,x_2)\cdot G^{(1,1)}(x_2,x_1) \\
&= |I_0|^2 + |G^{(1,1)}(x_1,x_2)|^2 = |I_0|^2(1+\cos^2\omega\tau)
\end{aligned} \tag{4}$$

where $x_1$and$x_2$represent two spatial points. As a result, the second order correlation function of the wave field consists of the first order serial correlation function and the square of the first order cross correlation function. Therefore, we can obtain the time difference τ from the intensity correlation of the wave field. And the intensity correlation degree indicates the value of the time differenceτ. For example, the maximum output of the intensity correlation function corresponds to the minimumτ, vice verse.

3. Measurement error constraint

Van Cittert-Zernicke theory gives the general result of the quasi-monochromatic incoherent extension source illuminating the complex coherence between two spatial points. The complex coherence between these two points is equal to the absolute value of the normalized Fourier transform of the intensity distribution function of the source [11,12]. It can be represented in formula (5), where $\Gamma$is the cross-correlation between light at P1 and P2. Z, r1, r2 represent the distance from Os to O, P1, P2 respectively, $\lambda$is the average wavelength, and $I(x_s, y_s)$ is the intensity distribution of the star surface.

$$\Gamma = \exp[i\frac{2\pi}{\lambda}(r_1 - r_2)] \times \frac{\iint_D I(x_s,y_s)\exp\{-i\frac{2\pi}{\lambda z}[(x_1-x_2)x_s + (y_1-y_2)y_s]\}dx_s dy_s}{\iint_D I(x_s,y_s)dx_s dy_s} \tag{5}$$