OPTIMAL PERIODIC MOTIONS OF TWO-MASS SYSTEMS IN RESISTIVE MEDIA

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Abstract: Progressive motions of two-mass systems in resistive media are analyzed. The motion control is implemented by means of periodic relative displacements of the masses. Different kinds of resistance forces acting upon the system are considered, including linear and nonlinear resistance depending on the velocity, as well as Coulomb's dry friction forces. Constraints are imposed on the relative displacements and velocities of the masses. Optimal periodic motions are determined that correspond to the maximal average speed of the system as a whole. Experimental data confirm the theoretical results obtained. Models of mobile mini-robots are described which are based on the principle presented in the paper.

Keywords: Multibody System, Periodic Motions, Periodic Control, Optimal Control, Mobile Robots

1. INTRODUCTION

A system of two or more bodies can move progressively in a resistive medium, if the bodies perform periodic motions relative to each other. One of these bodies (an inner one) can be contained within a certain closed cavity inside the other (outer) body, so that the system has no outward moving parts such as screws, wheels, legs, wings, etc. This well-known principle of motion is utilized in various projects of mobile robots and underwater vehicles (see, e.g., Breguet and Clavel, 1998; Schmoeckel and Worn, 2001; Vartholomeos and Papadopoulos, 2006).

In this paper, simple models of this phenomenon are analyzed. The mechanical system under consideration consists of two rigid bodies of masses M and m. For brevity, these bodies will be called body M and body m, respectively. Body m moves periodically relative to the main body M which interacts with the outward medium and is subject to resistance forces.

Different kinds of resistance forces acting upon body M are considered, including linear and nonlinear resistance depending on the velocity of the body, and also Coulomb's dry friction. The forces can be anisotropic, i.e., dependent on the direction of the velocity of body M.

The progressive motion of the system as a whole is controlled by the periodic motion of body m relative to body M. Simple relative periodic motions are analyzed, and constraints are imposed on the relative displacements and velocities. Under the constraints imposed, optimal parameters of the periodic motions are determined that correspond to the maximal average speed of the system as a whole. The results obtained (see also Chernousko

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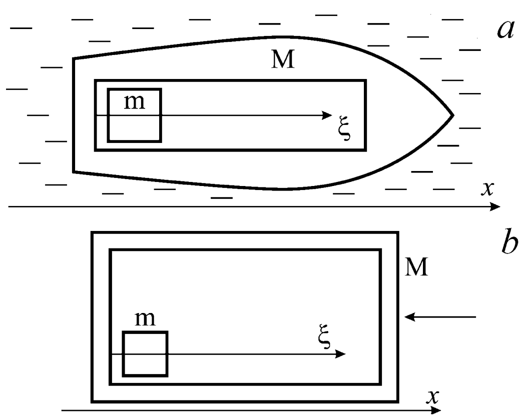

Fig. 1. Mechanical models

2005, 2006a,b) enable one to evaluate the maximal possible speed of mobile mechanical systems that utilize the principle of motion based on relative oscillations of parts of the system moving in a resistive medium.

Experimental results confirm the practical implementability of this principle of motion.

2. EQUATIONS OF MOTION

The system consists of two rigid bodies that can move along a straight line in a resistive medium (Fig. 1). Denote by x and v the absolute coordinate and velocity of the main body M, respectively, and by ξ, u, and w the displacement of the inner body m relative to body M, its relative velocity and acceleration, respectively.

The kinematic equations of motion of body m relative to body M are

$$\dot{\xi} = u, \qquad \dot{u} = w. \tag{1}$$

The dynamic equations for body M can be written as follows:

$$\begin{aligned} &\dot{x} = v, \qquad \dot{v} = -\mu w - r(v) \\ &\mu = m/(M+m), \end{aligned} \tag{2}$$

where $r(v)$ is the resistance force acting upon body M divided by the total mass of the system, $M + m$.

For the anisotropic linear resistance (Fig. 1a), the function $r(v)$ is given by

$$\begin{aligned} &r(v) = k_+ v, \quad \text{if} \quad v \geq 0; \\ &r(v) = k_- v, \quad \text{if} \quad v < 0. \end{aligned} \tag{3}$$

Similarly, for the anisotropic quadratic resistance, this function has the form

$$\begin{aligned} &r(v) = æ_+ |v| v, \quad \text{if} \quad v \geq 0; \\ &r(v) = æ_- |v| v, \quad \text{if} \quad v < 0. \end{aligned} \tag{4}$$

In Eqs. (3) and (4), k_+, k_-, $æ_+$, and $æ_-$ are positive coefficients. For the isotropic case, $k_+ = k_-$ and $æ_+ = æ_-$.

For the case of anisotropic Coulomb's friction (Fig. 1b), the function $r(v)$ is given by

$$\begin{aligned} &r(v) = f_+ g, \quad \text{if} \quad v > 0; \\ &r(v) = -f_- g, \quad \text{if} \quad v < 0, \end{aligned} \tag{5}$$

where g is the acceleration due to gravity, f_+ and f_- are coefficients of friction that can be different for onward and backward motions. If the inequalities

$$-f_- g \leq \mu w \leq f_+ g \tag{6}$$

hold and body M is at rest ($v = 0$), then it will stay at rest.

In what follows, the motion of body m relative to body M is supposed to be periodic with a period T and bounded within a fixed internal:

$$0 \leq \xi(t) \leq L, \tag{7}$$

where $L > 0$ is given. Without loss of generality, it is assumed that at the beginning and at the end of the period body m is at the left end of the internal, so that

$$\xi(0) = \xi(T) = 0, \qquad u(0) = u(T) = 0. \tag{8}$$

The maximal admissible displacement $\xi(0) = L$ is reached at some instant $\theta \in (0, T)$.

The motion of the system is controlled by the relative motion of body m, i.e., by functions $\xi(t)$, $u(t)$, and $w(t)$ subject to Eqs. (1) and conditions (7) and (8).

We will find the relative motions such that the velocity $v(t)$ of body M is T-periodic, i.e., $v(0) = v(T) = v_0$, and the average velocity of the system $V = \Delta x/T$, where $\Delta x = x(T) - x(0)$, is maximal.

3. LINEAR RESISTANCE

Note that the anisotropic resistance (3) is, in fact, nonlinear, if $k_+ \neq k_-$. In the case of the linear resistance, $k_+ = k_- = k$. Substitute $r(v)$ from Eq. (3) and w from Eq. (1) into (2) and integrate the resulting equation to obtain

$$v(T) - v(0) = -\mu[u(T) - u(0)] - k[x(T) - x(0)].$$

Since $u(t)$ and $v(t)$ should be T-periodic, the relation $x(T) = x(0)$ holds and, therefore, $V = 0$.