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A modified approach to estimate fractal dimension of gray scale images

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ABSTRACT

Fractal dimension (FD) is an important feature of fractal geometry to identify surface roughness of digital images. In this regard many methods were presented, among which differential box counting (DBC) method is a commonly used technique to estimate fractal dimension (surface roughness) of digital images. This paper presents modified version of differential box counting technique that addresses three issues found in original DBC; such as minimum roughness variation, computational error and similar fractal dimension (FD) evaluated either by incrementing or decrementing constant value to each intensity points. Based upon these three issues, our proposed method is better than the existing methods like DBC, relative DBC (RDBC) and improved DBC (IDBC). The improved version is achieved by subtracting the minimum intensity value from average intensity value on each grid. The subtraction of the minimum gray level of the block rather than zero gray level is used as a correction factor for accurate estimation of fractal dimension. The proposed methodology was demonstrated on real brodatz texture data base images, smooth images and synthetic texture like images. It shows that our improved method covers all objects with wider range of fractal dimension as compared to the existing methods.

1. Introduction

Fractal geometry has become most popular and useful technique for analysis of digital images in terms of its roughness. The image intensity surface can be treated as fractal object and characterises of these objects are evaluated mathematically by taking account of fractal dimension (FD). In real world, it may consist of maximum complex and irritated objects that cannot be measure through Euclidean geometry [1–5]. In order to evaluate the roughness of complex objects, the concept of fractal dimension comes into existence and it can be applied in several fields of application of image analysis in terms of shape measurement and classification [6], texture analysis and segmentation [7–9], and other field of graphics and image analysis [2,10]. The major property of fractal dimension is called self similarity and it is followed by several additional theories valid to a broad category of fractals in terms of gray scale and color images. Gangepain [11] proposed reticular cell counting method which was improved by Voss [12] associating probability theory. Next, Keller et al. [13,14] provides an additional refinement based upon the concept of linear interpolation. In this regard, several box counting mechanisms came into existence [1,11,15–18]. The box counting technique was given maximum preference due to its simplicity and compatibility.

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Based on the concept of box counting, Sarkar and Chaudhuri [15] studied 5 different algorithms of box counting technique on synthetic images and suggested an efficient differential box counting approach to computing fractal dimension of gray scale images. Their results have shown that Keller et al. [13,14] gives the satisfactory result, after reaching a particular level of noise (s), image intensity surface with the slope of the curve tends to zero. Here the estimate of fractal dimension (D) can be obtained from the least square linear fit of log (Nr) against log (1/r), which is described in later section. As the natural scenes rather exhibit some statistical self-similarity, not deterministic self-similarity, therefore both the methods does not estimate the dynamic range of FD; the reduction factor r was introduced so that if a scene is scaled down by a ratio r in all n dimensions, then it becomes statistically identical to the original one to satisfy Eq. (1) below. However, the differential box counting method was recognized as a better technique and was also supported by the investigations in many research papers [19,20]. In context to DBC approach, Nayak et al. [21] presented modified DBC approach by implementing asymmetric triangle box partition of square grid in order to optimize the performance of the method in terms of less fitting error and simultaneously provide more precision box count by means of triangle box partition. In gray scale domain, recently Nayak et al. [22] presented an experimental comparative analysis on different kinds of image sets using differential box counting and its improved version technique in order to indicate that the precise selection of FD technique is essential for accurate estimation of roughness of specific objects, and also they describe which method is most suitable for recognition accuracy.

The rest of the paper is organised as follows. In Section 2, we presented the basic fractal dimension estimation using DBC, RDBC and IDBC technique. Section 3 discuss about proposed methodology. Section 4 gives the experimental result and discussion. Section 5 presented regarding research discussion and finally concluding remarks are presented in Section 6.

2. Basic fractal dimension and various DBC approaches

2.1. Basic fractal dimension estimation

Fractal dimension or roughness of digital image can be estimated from the overall distribution of intensity points based on the concept of self-similarity. From the property of self-similarity we can say that the fractal is normally an irregular geometric structure that can be broken into smaller pieces; each smaller pieces is related to the original and similar to the original. A surrounded set X in Euclidean n-space is self-similar if X is the unification of Nr distinct (non-overlapping) copies of r itself scaled up or down by a factor of r. The fractal dimension D of X is given by Eq. (1).

$$D = \log(N_r)/\log(1/r) \tag{1}$$

Where Nr represents the distinct copies of X of the scale of reduction factor r. The union distinct copies of Nr should be completed fill of set X. We can only estimate the FD of deterministic fractals that means, the object having the property of deterministic self-similarity. DBC is most frequently used the algorithm for estimating fractal dimension of gray scale image. The details workflow of differential box-counting [15], Relative differential box-counting [20] and improved differential box counting method [24] are discussed below.

2.2. DBC approach

The DBC algorithm [15] considers an image of size $M \times M$ which has broken down of size$L \times L$, where L represents the box size of integer type of range between 2 to M/2. The image can be represented in 3D spatial space, where(x, y)representing 2D spatial space and 3rd coordinate Z representing gray level$G$. On the next step they partitioning (x, y) plane into grids of size$L \times L$, on every grid contains column of boxes of size$L \times L \times L'$, where L' represents height of the box and can be evaluated as$L' = L * G/M$ and reduction factor r can be computed asM/L. Let the minimum and maximum gray level of input image fall into respectively k^{th} and L^{th} then $n_r(i, j)$ can be evaluated as follows:

$$n_r(i, j) = L - K + 1 \tag{2}$$

Finally N_r can compute by taking contribution from each grid of scale r based on Eq. (3).

$$N_r = \sum_{i,j} n_r(i, j) \tag{3}$$

2.3. RDBC approach

Based on original DBC, Jin et al. [20] presented an improved version of DBC called relative DBC (RDBC)) by adopting same maximum and minimum intensity point on the grid and taking the scale limit such as upper and lower limits of scale ranges for accurate FD estimation of texture images. Finally Nr can compute by taking contribution from each grid of scale r as follows:

$$N_r = \sum_{i,j} ceil[k * ((\max(i, j) - \min(i, j)/L') \tag{4}$$

Where k represents the coefficient in z-direction and $ceil(.)$is used to set the nearest integer.