Optik 161 (2018) 180–186

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Original research article

Optical solitons in parabolic law medium with weak non-local nonlinearity using modified extended direct algebraic method

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ARTICLE INFO

Article history:
Received 15 January 2018
Accepted 9 February 2018

OCIS:
060.2310
060.4510
060.5530
190.3270
190.4370

Keywords:
Solitons
Non-local nonlinearity
Modified extended direct algebraic method

ABSTRACT

This paper obtains bright and dark-singular combo optical solitons in a parabolic law medium that is coupled with weak non-local nonlinearity. The method of modified extended direct algebraic method is applied to secure these soliton solutions. Additionally, several other solutions in terms of elliptic functions naturally fall out of the integration scheme as a byproduct.

1. Introduction

Optical solitons is one of the most thriving and revealing areas of research in the field of nonlinear optics. There are many exciting results that have been reported during the past few decades. The main governing model is the nonlinear Schrödinger's equation (NLSE) that has been studied with various nonlinear forms. In fact, there are a variety of mathematical phenomena, both analytical and numerical, that have been successfully applied to NLSE and other nonlinear evolution equations to retrieve solitons and other solutions [1–15]. This paper will study NLSE with a different form of nonlinearity

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https://doi.org/10.1016/j.ijleo.2018.02.030
0030-4026/

that is fairly new in the literature although it has been visible for the past few years. It is the combination of parabolic law nonlinearity and non-local nonlinear medium [2–5,14,15]. The modified direct extended algebraic method secures soliton and other solutions to the governing model. Thus, bright and dark-singular combo optical soliton solutions are obtained amongst various other solutions. The integration algorithm is initially reviewed followed with its detailed application to our model of study.

1.1. Governing model

The aim of the present work is to study the dynamics of optical solitons in a medium with competing weakly nonlocal nonlinearity and parabolic law nonlinearity. Without any loss of generality, the dimensionless nonlinear model is given by [15,14]

$$i\frac{\partial u}{\partial z}+a\frac{\partial^2 u}{\partial x^2}+b\frac{\partial^2(\mid u\mid^2 u)}{\partial x^2}u+c\mid u\mid^2 u+\mu\mid u\mid^4 u=0, \tag{1}$$

where a represents the coefficients of group velocity dispersion, while b gives the coefficient of weakly nonlocal nonlinearity, and finally the terms with c and μ account for the parabolic law (cubic–quintic) nonlinearity [15,14,10,11].

1.2. Overview of modified extended direct algebraic method

We suppose that the nonlinear evolutio equation for $u(x, t)$ to be of the form

$$P(u, u_t, u_x, u_{tx}, u_{tt}, u_{xx}\ldots)=0, \tag{2}$$

where P is a polynomial in its arguments. The essence of the modified extended direct algebraic method can be presented in the following steps [6,1,9,8,7,12]:

Step-1: Seek traveling wave solutions of Eq. (2) by taking $u(x, t) = U(\xi)$, $\xi = kx - \omega t$ and transform Eq. (2) to the ordinary differential equation

(3)$Q(U, U', U'', \ldots)=0$,where primes denote the derivative with respect to ξ.

Step-2: We introduce the solution $U(\xi)$ of Eq. (3) in the finite series form [6,1,9,8,7,12]

$$U(\xi)=\sum_{i=-N}^{N} a_i\phi(\xi)^i, \tag{4}$$

where a_i are real constants with $a_N \neq 0$ to be determined, N is a positive integer to be determined. $\phi(\xi)$ express the solution of the following equation [7]:

$$\phi'(\xi)=\sqrt{c_0+c_1\phi(\xi)+c_2\phi^2(\xi)+c_3\phi^3(\xi)+c_4\phi^4(\xi)+c_5\phi^5(\xi)+c_6\phi^6(\xi)}, \tag{5}$$

where c_i are constants and can be discussed as in [13].

Step-3: Determine N. This, usually, can be accomplished by balancing the linear term(s) of highest order with the highest order nonlinear term(s) in Eq. (3).

Step-4: Substituting Eq. (4) together with Eq. (5) into Eq. (3) yields an algebraic equation involving powers of $\phi(\xi)$. Equating the coefficients of each power of $\phi(\xi)$ to zero and discussing the value of c_i[13] gives a system of algebraic equations for a_i. Then, we solve the system with the aid of a computer algebra system (CAS), such as Mathematica or Maple, to determine these constants. On the other hand, depending on the value of parameters c_i[13], the solutions of Eq. (3) are well known to us. So, as a final step, we can obtain exact solutions of the given Eq. (1) [13].

2. Application of the modified extended direct algebraic method

Assume that Eq. (1) admits the following stationary solutions:

$$u(x, z)=\psi(x)e^{i\lambda z}, \tag{6}$$

where λ is the propagation constant. Substituting the above hypothesis Eq. (6) into Eq. (1) yields

$$(a+2b\psi^2)\psi_{xx}+2b\psi\psi_x^2-\lambda\psi+c\psi^3+\mu\psi^5=0. \tag{7}$$

Balancing the highest order derivative $\psi^2\psi_{xx}$ and nonlinear term ψ^5, we find $N = 1$. Consequently we reach

$$\psi(x)=a_{-1}\left(\phi(\xi)\right)^{-1}+a_0+a_1\phi(\xi). \tag{8}$$

Substituting Eq. (8) and Eq. (5) into Eq. (7) and using [13], the solutions of Eq. (1) can be expressed as follows:

Case 1. $c_0 = c_1 = c_3 = c_5 = c_6 = 0,\ a_{-1}=0,\ a_0=0,\ a_1=\sqrt{-\frac{2ac_4}{4bc_2+c}},\ \lambda=ac_2,\ \mu=3\frac{b(4bc_2+c)}{a}$.