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Extreme nonlinear waves in external gravitational-like potentials: Possible applications for the optical soliton supercontinuum generation and the ocean coast line protection

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ABSTRACT

In view of the fact that the nonlinear Schrödinger equation model has a wide range of applicability, from nonlinear femtosecond optics, Bose–Einstein condensates and plasmas, to ocean extreme (monster) waves and hurricanes, we conclude that existing mathematical analogies open advantageous possibility to study optical, matter-wave and hydrodynamic nonlinear waves and solitons in parallel and due to the evident complexity of experiments with extreme nonlinear waves in oceans, offer remarkable potentialities in studies of these so complex systems by performing "optic hydrodynamics" experiments in the nonlinear fiber-optics systems. Specifically, we present the results of computer experiments of soliton interactions with different external gravitational-like potentials including that with varying levels of inclinations. We demonstrate novel effective mechanism of the supercontinuum generation both in nonlinear optics and in hydrodynamics based on the higher order N-soliton wave packets decay induced by external potentials of the hyperbolic tangent profiles with varying inclinations similar to the real ocean coast line profiles.

1. Introduction

The interpenetration of the main ideas and methods being used in different fields of science and technology has become today one of the decisive factors in the progress of science as a whole. Among the most spectacular examples of such an interchange of ideas, analogies, and theoretical methods for analysis of totally different physical phenomena is the problem of the so-called extreme (monster) nonlinear waves formation arising in the framework of the nonlinear Schrödinger equation model (NLSE). The NLSE model has a wide range of applicability, from nonlinear femtosecond optics, Bose–Einstein condensates (BECs) and plasmas, to ocean monster (rogue) waves and hurricanes, and has brought together scientists from

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vastly different areas, in particular, from atomic, low-temperature and condensed matter physics, ultrashort nonlinear optics and lasers, fluid mechanics and fundamental general and particle physics. Different applications of the soliton concept including dynamics of solitons in external potentials, nonautonomous solitons and the search of exactly integrable models are now under active study [1–21].

It is well known fact that, from the mathematical point of view, the following one-dimensional NLSE models arising in nonlinear optics for the complex envelope of the electric field $E(x, t)$ both for temporal and spatial optical solitons

$$i\frac{\partial E}{\partial x} = -\frac{1}{2}\frac{\partial^2 k}{\partial \varpi^2}\frac{\partial^2 E}{\partial t^2} + k_0\frac{n_{2eff}}{n_0}|E|^2 E + U_{\text{ext}}(x, t)E, \tag{1}$$

$$2ik_0\frac{\partial E}{\partial z} = \frac{\partial^2 E}{\partial x^2} + \frac{2k_0^2 n_2}{n_0}|E|^2 E + U_{\text{ext}}(x, t)E, \tag{2}$$

the Gross–Pitaevskii equation in BECs for the matter wave-function $\Phi(x, t)$

$$i\hbar\frac{\partial}{\partial t}\Phi(x,t) = -\frac{\hbar^2}{2m}\frac{\partial^2 \Phi}{\partial x^2} + \frac{2\hbar^2 a(t)}{m a_{ho\perp}^2} \mid \Phi(x,t) \mid^2 \Phi(x,t) + U_{\text{ext}}(x,t)\Phi(x,t), \tag{3}$$

and the deep water NLSE model for the complex envelope $A(x, t)$ of the corresponding wave train (so that the surface elevation is determined by the real part of the function $A(x, t)$

$$i\left(\frac{\partial A}{\partial t} + \frac{\varpi_0}{2k_0}\frac{\partial A}{\partial x}\right) = \frac{\varpi_0}{8k_0^2}\frac{\partial^2 A}{\partial x^2} + \frac{\varpi_0 k_0^2}{2}|A|^2 A + U_{\text{ext}}(x, t)A. \tag{4}$$

are similar. In particular, Eqs. (1)–(3) provide explicit predictions about the dynamics of laser pulses in optical fibers and gravity waves in one-dimensional water channels [1–21]. All Eqs. (1)–(4) are written here in the standard notations for each field of applications (see, for instance, [19–23] and references therein), and in the most interesting case of the linear gravitational-like external potential they are mathematically similar to the Chen and Liu nonisospectral model, in general, with varying in time spectral parameter $\lambda(t)$ [24]

$$i\frac{\partial q}{\partial t} + \frac{1}{2}\frac{\partial^2 q}{\partial x^2} + |q|^2 q - 2\lambda(t)xq = 0. \tag{5}$$

The key feature of the NLSE model (5) consists in the fact that this model remains to be exactly integrable, so that the gravitational-like linear potentials (considered here as analogous to "the ideal linear coastal line profiles") cannot radically alter the soliton dynamics and do not destroy the solitons and all exact solutions of the completely integrable NLSE model including the rogue (monster) waves.

Both in nonlinear optics and hydrodynamics, the main features of optical soliton supercontinuum and the hydrodynamic supercontinuum were shown to be associated with the decay (fission) of the initial higher-order NLSE solitons into individual fundamental solitons due to the higher-order nonlinear perturbations to the governing extended NLSE models [14,25].

The central idea of our work is to demonstrate another possible way of looking at the additional physical mechanism of the higher-order NLSE solitons decay and supercontinuum generation both in nonlinear optics and in hydrodynamics. In the first part of the paper, we present the review of the main features of linear and nonlinear accelerating wave-packets. In the second part, we present the detailed studies of the NLSE solitons interactions with different external potentials including that with varying levels of inclinations, and demonstrate the effective mechanism of the break-up both of the fundamental soliton and the higher order N-soliton wave packets. Based on a wide range of applicability of the NLSE model, we conclude that considered mathematical analogies open the possibility to study optical, matter-wave and hydrodynamic solitons in parallel and due to the evident complexity of experiments with matter-wave solitons and extreme nonlinear waves in oceans, offer remarkable possibilities in studies of these so complex systems by performing experiments in the nonlinear fiber-optics systems.

2. Linear and nonlinear accelerating wave-packets in gravitational-like potentials

Historically, the discoveries of the accelerating linear and nonlinear wave-packets in the linear gravitational-like potentials are exceptionally intriguing. The discovery of linear accelerating matter-wave packet dates back to the early days of quantum mechanics [26]. Interestingly, that in parallel, the discovery of nonlinear accelerating wave packet (envelope soliton) also dates back to the first days of the development of the Inverse Scattering Transform (IST) method with varying in time spectral parameter [24].

In quantum mechanics, the linear Schrödinger equation (SE)

$$i\hbar\frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} + mgx\Psi, \tag{6}$$