Optik 160 (2018) 33–43

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Original research article

Optical soliton perturbation with resonant nonlinear Schrödinger's equation having full nonlinearity by modified simple equation method

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ARTICLE INFO

Article history:
Received 3 January 2018
Accepted 27 January 2018

OCIS:
060.2310
060.4510
060.5530
190.3270
190.4370

Keywords:
Solitons
Perturbation
Full nonlinearity
Non-Kerr law

ABSTRACT

This paper retrieves soliton solutions to the perturbed resonant nonlinear Schrödinger's equation with time-dependent coefficients using the modified simple equation method. The four types of exotic non-Kerr laws studied in this paper are quadratic–cubic law, anti-cubic law, cubic–quintic–septic law and triple-power law. These soliton solutions appear with constraint conditions that guarantee their existence.

1. Introduction

Optical soliton perturbation is one of the fastest growing areas of research in the field of telecommunications engineering. There is an ever increasing demand for electronic communications system with the advent of modern social media and other such communication means. The soliton molecule transmission through optical fibers, metamaterials, PCF and other such form of waveguides can meet this growing demand. Therefore, it is imperative to venture further into this exciting field that will lead to novel results.

This paper studies resonant solitons in the context of nonlinear optics with some exotic non-Kerr law nonlinearities that are less visible in this context. There are several mathematical approaches that have been successfully applied all across in this arena to extract solitons and additional solutions [1–15]. This paper implements the modified simple equation method

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https://doi.org/10.1016/j.ijleo.2018.01.098

to address the governing resonant nonlinear Schrödinger's equation (RNLSE) to secure soliton solutions that will be an asset in the literature of soliton dynamics. Dark and singular soliton solutions will be revealed along with their existence criteria that naturally emerge from the solution structure. As a byproduct of this scheme, singular periodic solutions also fall out that are not considered in the telecommunications industry. The integration algorithm and the solution spectrum are all detailed in the next couple of sections.

1.1. Governing model

The RNLSE with time-dependent coefficients is given by

$$iq_t + \alpha(t) q_{xx} + \beta(t) F\left(|q|^2\right) q + \gamma(t) \left(\frac{|q|_{xx}}{|q|}\right) q = 0. \tag{1}$$

Here, in (1), the first term is the linear evolution, while $\alpha(t)$ is the coefficient of group velocity dispersion (GVD) and $\beta(t)$ is the coefficient of nonlinearity. Finally, $\gamma(t)$ is quantum or Bohm potential that appears in the context of chiral solitons inquantum Hall effect. It is also seen in the context of Madelung fluid in quantum mechanics. Also, the functional F meets the following technical criteria:

F is a real-valued algebraic function and it is necessary to have the smoothness of the complex function $F\left(|q|^2\right) q : C \to C$. Considering the complex plane C as a two-dimensional linear space R^2, the function $F\left(|q|^2\right) q$ is k times continuously differentiable, so that

$$F\left(|q|^2\right) q \in \bigcup_{m,n=1}^{\infty} C^k((-n, n) \times (-m, m); \quad R^2).$$

In presence of perturbation terms with time-dependent coefficients, RNLSE extends to:

$$iq_t + \alpha(t) q_{xx} + \beta(t) F\left(|q|^2\right) q + \gamma(t) \left(\frac{|q|_{xx}}{|q|}\right) q = i\left[\delta(t) q_x + \lambda(t)\left(|q|^{2m} q\right)_x + \mu(t)\left(|q|^{2m}\right)_x q\right] \tag{2}$$

where $\delta(t)$ is the inter-modal dispersion, $\lambda(t)$ represents the coefficient of self-steepening for short pulses and $\mu(t)$ is the higher-order dispersion coefficient. The parameter m accounts for full nonlinearity.

2. Quick overview of modified simple equation method

Suppose we have a nonlinear evolution equation in the form:

$$P(u, u_t, u_x, u_{tt}, u_{xt}, u_{xx}, \ldots) = 0 \tag{3}$$

where P is a polynomial in $u(x, t)$ and its partial derivatives in which the highest order derivatives and nonlinear terms are involved. In the following, we enumerate the essential steps of this algorithm.

Step-1: We start with the transformation

$$u(x, t) = U(\xi), \quad \xi = x - ct, \tag{4}$$

where c is a constant that needs to be determined, to reduce Eq. (3) to the following ordinary differential equation:

$$Q\left(U, U', U'', U''', \ldots\right) = 0 \tag{5}$$

where Q is a polynomial in $U(\xi)$ and its total derivatives, while $' = d/d\xi$.

Step-2: We assume that Eq. (5) has the formal solution

$$U(\xi) = \sum_{l=0}^{N} a_l \left(\frac{\psi'(\xi)}{\psi(\xi)}\right)^l, \tag{6}$$

where a_l are constants to be determined, such that $a_N \neq 0$, and $\psi(\xi)$ is an unknown function to be determined later.

Step-3: The positive integer N in Eq. (6) is determined by considering the homogeneous balance between the highest order derivatives and the nonlinear terms in Eq. (5).

Step-4: We substitute (6) into (5) and calculate all the necessary derivatives U',U'',. . .of the unknown function $U(\xi)$ and we account the function $U(\xi)$. As a result of this substitution, we get a polynomial of $\psi'(\xi)/\psi(\xi)$ and its derivatives. In this polynomial, we collect all terms with like powers of $\psi^{-j}(\xi)$, j = 0,1,2,. . .and its derivatives, and subsequently we equate to zero all the coefficients of this polynomial. This operation yields a system of equations which can be solved to find a_k and $\psi(\xi)$. This leads to the retrieval of exact solutions for Eq. (3).