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Optical soliton perturbation with complex Ginzburg–Landau equation using trial solution approach

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ABSTRACT

This paper retrieves optical soliton solution to the perturbed complex Ginzburg–Landau equation that is studied with nine different forms of nonlinearity. The trial solutions approach is the integration algorithm adopted in this paper. The perturbation terms appear with full nonlinearity to get a taste of generalized setting. Bright, dark and singular soliton solutions are obtained. The existence criteria of such solitons are also presented.

1. Introduction

Optical soliton perturbation and in general is one of the most fascinating areas of research in the field of mathematical photonics. There are several models studied in this context in various areas of mathematical physics [1–15]. The most fundamental and widely visible model is the nonlinear Schrödinger's equation. There are several models that stem out of it. They are Chen–Lee–Liu equation, Sasa–Satsuma model, Gerdjikov–Ivanov equation, Lakshmanan–Porsezian–Daniel model, Schrödinger–Hirota equation and a variety of other such models. All of them describe the dynamics of soliton propagation through optical fibers and other forms of waveguides under different circumstances. This paper will study one such model

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that is the complex Ginzburg–Landau equation (CGLE). It is perturbed version will be considered in this paper where the perturbation terms are all Hamiltonian type and appear with full nonlinearity for a generalized flavor. There are nine different types of nonlinear media studied. The integration scheme adopted in this paper is known as trial solution method. This approach to integration has been succesfully applied to various other optical devices such as couplers, metamaterials and polarization–preserving fibers [3,4,15]. Additionally, this scheme also was fruitfully applied to the unperturbed CGLE, in the past [12]. Today, this algorithm will extract bright, dark and singular soliton solutions to perturbed CGLE. These soliton solutions will be possible for parameter restrictions that are also presented in the paper.

1.1. Model equation

The dimensions form of CGLE is as follows [6–8,12–14]:

$$iq_t + aq_{xx} + bF\left(|q|^2\right)q = \frac{1}{|q|^2 q^*}\left[\alpha |q|^2\left(|q|^2\right)_{xx} - \beta\left\{\left(|q|^2\right)_x\right\}^2\right] + \gamma q \tag{1}$$

where x represents the non-dimensional distance along the fibers, while t represents time in dimensionless form; a, b, α, β and γ are valued constants. The coefficients a and b come from the group velocity dispersion (GVD) and nonlinearity, respectively. The terms with α, β and γ arise from the perturbation effects in particular, γ comes from the detuning effect.

In (1), F is real-valued algebraic function and it is necessary to possess the smoothness of the complex function $F\left(|q|^2\right)q$ is k times continuously differentiable, so that

$$F\left(|q|^2\right)q \in \bigcup_{m,n=1}^{\infty} C^k((-n,n) \times (-m,m);\ R^2).$$

In presence of perturbation terms, CGLE is modified to

$$iq_t + aq_{xx} + bF\left(|q|^2\right)q = \frac{1}{|q|^2 q^*}\left[\alpha |q|^2\left(|q|^2\right)_{xx} - \beta\left\{\left(|q|^2\right)_x\right\}^2\right] + \gamma q + i\left[\delta q_x + \lambda\left(|q|^{2m}q\right)_x + \mu\left(|q|^{2m}\right)_x q\right] \tag{2}$$

where δ is the inter-modal dispersion, λ represents the coefficient of self-steepening for short pulses and μ is the higher-order dispersion coefficient. The parameter m is responsible for full nonlinearity.

2. A quick glance at trial equation method

In this section we outline the main steps of the trial equation method as following:

Step-1: Suppose a nonlinear PDE with time-dependent coefficients

$$P(u, u_t, u_x, u_{tt}, u_{xt}, u_{xx}, \ldots) = 0 \tag{3}$$

can be converted to an ordinary differential equation (ODE)

$$Q\left(U, U', U'', U''', \ldots\right) = 0 \tag{4}$$

using a travelling wave hypothesis $u(x, t) = U(\xi)$, $\xi = x - vt$, where $U = U(\xi)$ is an unknown function, Q is a polynomial in the variable U and its derivatives. If all terms contain derivatives, then Eq. (4) is integrated where integration constants are considered zeros.

Step-2: Take the trial equation

$$\left(U'\right)^2 = F(U) = \sum_{l=0}^{N} a_l U^l \tag{5}$$

where a_l ($l = 0, 1, \ldots, N$) are constants to be determined. Substituting Eq. (5) and other derivative terms such as U'' or U''' and so on into Eq. (4) yields a polynomial $G(U)$ of U. According to the balance principle we can determine the value of N. Setting the coefficients of $G(U)$ to zero, we get a system of algebraic equations. Solving this system, we can determine v and values of $a_0, a_1, \ldots, a_N$.

Step-3: Rewrite Eq. (5) by the integral form

$$\pm\left(\xi - \xi_0\right) = \int \frac{dU}{\sqrt{F(U)}} \tag{6}$$

According to the complete discrimination system of the polynomial, we classify the roots of $F(U)$, and solve the integral Eq. (6). Thus we obtain the exact solutions to Eq. (3).