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Optical solitons and group invariant solutions to Lakshmanan–Porsezian–Daniel model in optical fibers and PCF

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ABSTRACT

The Lie symmetry analysis is applied to Lakshmanan–Porsezian–Daniel model, with Kerr and power law nonlinearity, to retrieve optical soloiton solutions. Singular soliton solutions are available by the application of this algorithm.

1. Introduction

The theory of soliton propagation through optical fibers, metamaterials and PCF is governed by the well-known nonlinear Schrödinger's equation that comes with several forms of nonlinearity. While this is the most visible model to address soliton dynamics in nonlinear optics, several other models have subsequently sprung up with time and they have also gained a fair amount of popularity over the years. One such equation is the Lakshmanan–Porsezian–Daniel (LPD) model that has been extensively and successfully studied by various authors all across the globe. There are several mathematical tools and algorithms available and these have been implemented to address optics problems as well as other nonlinear evolution equations (NLEEs) [1–15]. This paper implements one of the most powerful mathematical tool to study the model. It is Lie

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symmetry analysis. This scheme reveals soliton solutions to the LPD model with Kerr and power laws of nonlinearity through group invariance. The rest of the paper details the methodology after a quick introduction to the governing equation.

1.1. Governing model

The dimensionless form of LPD model with higher order dispersion, full nonlinearity and spatio-temporal dispersion to be analysed here for symmetries and optical solitons is given by [1,3,4,8,9,11,13]

$$\begin{aligned} &iq_t + aq_{xx} + bq_{xt} + cF\left(|q|^2\right)q = \\ &\sigma q_{xxxx} + \alpha (q_x)^2 q^* + \beta |q_x|^2 q + \gamma |q|^2 q_{xx} + \lambda q^2 q^*_{xx} + \delta |q|^4 q, \end{aligned} \tag{1}$$

where $q(x, t)$ is a complex valued wave function with the independent variables being x and t that represents space and time respectively. The first term in (1) represents linear temporal evolution while a and b are coefficients of group velocity dispersion (GVD) and spatio-temporal dispersion (STD) respectively. During 2012, it was pointed out that GVD alone makes the model ill-posed and therefore the inclusion of STD was proposed [6,10]. The coefficient of c in (1) on its left hand side is the nonlinear term where the functional F stands for the two kinds of nonlinearity that will be studied in this paper, which are Kerr law (or cubic law) and power law. Eq. (1) will now be addressed using Lie symmetry analysis in the next section.

2. Classical Lie symmetry analysis

The study of symmetry analysis for NLEEs, has gained popularity in the fields of mathematics and physics, such as locating symmetries, symmetry groups of transformation, symmetry reductions and construction of group invariant solutions [2,5,7,12,14,15]. In this work, we will use Lie classical method to investigate the symmetry reductions of the LPD Eq. (1) for both Kerr law and power law nonlinearity.

2.1. Kerr law

For Kerr law, the functional is described by

$$F(u) = u \tag{2}$$

which transforms the LPD model to be

$$iq_t + aq_{xx} + bq_{xt} + c|q|^2 q = \sigma q_{xxxx} + \alpha (q_x)^2 q^* + \beta |q_x|^2 q + \gamma |q|^2 q_{xx} + \lambda q^2 {q_{xx}}^* + \delta |q|^4 q, \tag{3}$$

First of all, we take the complex function $q(x, t)$ as

$$q(x, t) = u(x, t) + iv(x, t), \tag{4}$$

which decomposes Eq. (3) into the following system of equations:

$$\begin{aligned} &-v_t + au_{xx} + bu_{xt} + c(u^2 + v^2)u = \sigma u_{xxxx} + \alpha (u{u_x}^2 - u{v_x}^2 + 2u_x v_x v) \\ &+\beta u({u_x}^2 + {v_x}^2) + \gamma (u^2 + v^2)u_{xx} + \lambda (u^2 u_{xx} - v^2 u_{xx} + 2uv v_{xx}) + \delta (u^2 + v^2)^2 u, \end{aligned} \tag{5}$$

and

$$\begin{aligned} &u_t + av_{xx} + bv_{xt} + c(u^2 + v^2)v = \sigma v_{xxxx} + \alpha (v{v_x}^2 - v{u_x}^2 + 2u_x v_x u) \\ &+\beta v({u_x}^2 + {v_x}^2) + \gamma (u^2 + v^2)v_{xx} + \lambda (v^2 v_{xx} - u^2 v_{xx} + 2uv u_{xx}) + \delta (u^2 + v^2)^2 v. \end{aligned} \tag{6}$$

To find the symmetries, let us consider the Lie group of point transformations as

$$\begin{aligned} &u^* = u + \epsilon \eta(x, t, u, v) + O(\epsilon^2), \\ &v^* = v + \epsilon \phi(x, t, u, v) + O(\epsilon^2), \\ &x^* = x + \epsilon \xi(x, t, u, v) + O(\epsilon^2), \\ &t^* = t + \epsilon \tau(x, t, u, v) + O(\epsilon^2), \end{aligned} \tag{7}$$