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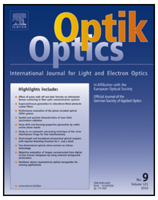

Original research article

Chirped solitons in optical metamaterials with parabolic law nonlinearity by extended trial function method

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ABSTRACT

This paper secures chirped bright and singular soliton solutions in an optical metamaterial that is considered with parabolic law nonlinearity and self-steepening effect. The extended trial function method is the integration scheme adopted in this paper. Several other solutions such as singular periodic waves and elliptic functions naturally emerge as a byproduct of this scheme.

1. Introduction

There are various kinds of waveguides that support optical soliton propagation for trans-continental and trans-oceanic distances. This makes the world of telecommunications an engineering marvel. Such waveguides are optical fibers, PCF, waveguides with rib and ring geometry, laser-induced waveguides, slab waveguides, strip waveguides and various others. This paper will address soliton propagation thgrough a newly proposed form of waveguide that is known as optical metamaterial. There are several results that have been reported with this kind of waveguide dynamics. Several forms of mathematical architecture have been implemented to study the dynamics of solitons in various waveguides as well as in other areas of soliton theory [1–20]. This paper studies soliton propagation with parabolic law nonlinearity through an optical metamaterial that is considered with parabolic law nonlinearity with self-steepening effect which is treated as a strong perturbation term. The governing model is the nonlinear Schrödinger's equation (NLSE). The study is conducted with extended trial function scheme that successfully retrieves bright and singular soliton solutions to the model.

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2. Theoretical model

The NLSE describing the propagation of ultrashort pulses through optical metamaterials governed by parabolic-law nonlinearity and self-steepening effect is written in the following form [12,14]:

$$i\frac{\partial\psi}{\partial z}+\frac{k_2}{2}\frac{\partial^2\psi}{\partial t^2}+p_3|\psi|^2\psi-p_5|\psi|^4\psi-is_1\frac{\partial}{\partial t}\left(|\psi|^2\psi\right)=0, \tag{1}$$

where $\psi(z, t)$ is the complex envelope of the electric field, $t = cT/\lambda_p$ and $z = Z/\lambda_p$ are the respective normalized time and propagation distance, where λ_p is the plasma wavelength. k_2 stands for the group-velocity dispersion (GVD), while p_3, p_5 and s_1 denote the cubic, pseudo-quintic nonlinearity and the self-steepening effect, respectively.

2.1. Mathematical analysis

To start, we search for the complex envelope traveling-wave solutions to the governing model (1) in the form [1,6–11,17]:

$$\psi(z,t)=\rho(\xi)e^{i[\chi(\xi)-kz]}, \tag{2}$$

where $\xi = t - uz$ is the traveling coordinate, while ρ and χ are real functions of the traveling coordinate ξ. Here $u = 1/v$, with v the group velocity of the wave packet. The corresponding chirp is given by

$$\delta\omega(t,z)=-\frac{\partial}{\partial t}[\chi(\xi)-kz]=-\chi'(\xi). \tag{3}$$

Substituting (2) into (1) and separating the real and imaginary parts, we get the following set of equations:

$$k\rho+u\chi'\rho-\frac{k_2}{2}\chi'^2\rho+\frac{k_2}{2}\rho''+s_1\chi'\rho^3+p_3\rho^3-p_5\rho^5=0, \tag{4}$$

and

$$-u\rho'+\frac{k_2}{2}\rho\chi''+k_2\rho'\chi'-3s_1\rho^2\rho'=0. \tag{5}$$

To solve Eqs. (4) and (5), we adopt the ansatz:

$$\chi'=p\rho^2+q, \tag{6}$$

where p and q are the nonlinear and constant chirp parameters, respectively. Accordingly, the resultant chirp takes the form:

$$\delta\omega(t,z)=-(p\rho^2+q). \tag{7}$$

This implies that the chirp associated with propagating pulses is intensity dependent and includes both linear and nonlinear contributions. Further substitution of the *ansatz* (6) into Eq. (5) yields the relations of q and p as

$$p=\frac{3s_1}{2k_2},\quad q=\frac{u}{k_2}. \tag{8}$$

As seen from the first relation of (8), the nonlinear chirp parameter depends on the self-steepening coefficient s_1. Therefore, we can conclude that the origin of the nonlinear chirp comes from the higher-order nonlinear effects such as the self-steepening effect.

Now, employing Eqs. (6) and (8) in Eq. (4), the following elliptic differential equation is obtained:

$$\rho''+\delta\rho+\beta\rho^3+\gamma\rho^5=0, \tag{9}$$

where

$$\delta=\frac{2k_2k+u^2}{k_2^2},\quad \beta=\frac{2(k_2p_3+us_1)}{k_2^2},\quad \gamma=\frac{3s_1^2-8k_2p_5}{4k_2^2}. \tag{10}$$

Balancing ρ'' with ρ^5 in Eq. (9) yields

$$N+2=5N, \tag{11}$$

so that

$$N=\frac{1}{2}. \tag{12}$$

In order to acquire a closed form analytic solution, we utilize a transformation formula

$$\rho=U^{\frac{1}{2}}. \tag{13}$$