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Optical solitons with differential group delay by trial equation method

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ABSTRACT

This paper secures bright, dark and singualr soliton solutions in birefringent fibers. Both Kerr law and parabolic law nonlinearity are studied. The trial equation method successfully retrieves these solitons with constraint relations that assure their existence.

1. Introduction

The propagation of soliton molecules through optical fibers has drawn lots of attention in telecommunication industry. Thus, modern day communication technology is not possible without a thorough understanding of the soliton dynamics. One of the disadvantages of such long-haul communication of optical solitons is the effect of pulse splitting that arises naturally due to several factors. These include rough handling of optical fibers along with its bend and twist as well as other such unwanted features that naturally arise. These lead to differential group delay and its cumulative effect gives rise to birefringence in optical fibers. Therefore, such a feature in optical fibers must be independently addressed.

There has been several mathematical techniques that has been implemented to study these birefringent fibers [1–10]. These include the method of undetermined coefficients [1,3], Lie symmetry analysis [8], extended trial equation method [4] and several others. This paper will address birefringent fibers with Kerr and parabolic law nonlinearity with the inclusion of a few Hamiltonian perturbation terms that do not destroy the integrability aspect of the model equation. Our integration

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technique in this paper will be trial equation method which will yield bright, dark and singular soliton solutions in such fibers. The existence of such solitons are assured with listed constraint conditions. After a quick review of the integration scheme, the details of the derivation of soliton solutions are discussed in the subsequent section.

2. Revisitation of trial equation method

In this section we outline the main steps of the trial equation method as:

Step-1: Suppose a nonlinear evolution equation with time-dependent coefficients is represented as:

$$P(u, u_t, u_x, u_{tt}, u_{xt}, u_{xx}, \ldots) = 0 \tag{1}$$

can be converted to an ordinary differential equation ODE

$$Q(U, U', U'', U''', \ldots) = 0 \tag{2}$$

using a travelling wave hypothesis $u(x, t) = U(\xi)$, $\xi = x - vt$, where $U = U(\xi)$ is an unknown function, Q is a polynomial in the variable U and its derivatives. If all terms contain derivatives, then Eq. (2) is integrated where integration constants are taken to be zero without any loss of generality since soliton solutions are targeted.

Step-2: Take the trial equation

$$(U')^2 = F(U) = \sum_{l=0}^{N} \delta_l U^l \tag{3}$$

where δ_l, $(l = 0, 1, \ldots, N)$ are unknown constants that are yet to be determined. Substituting Eq. (3) and other derivative terms such as U'' or U''' and so on into Eq. (2) yields a polynomial $G(U)$ of U. According to the balance principle we can determine the value of N. Upon setting the coefficients of $G(U)$ to zero, we uncover a system of algebraic equations. Once this system is solved, we recover v and values of $\delta_0, \delta_1, \ldots, \delta_N$.

Step-3: Rewrite Eq. (3) in the integral form as

$$\pm(\xi - \xi_0) = \int \frac{dU}{\sqrt{F(U)}} \tag{4}$$

Based on the discriminants of the polynomial, we classify the roots of $F(U)$, and subsequently solve the integral Eq. (4). Thus we can locate exact solutions to Eq. (1).

3. Application to birefringent fibers

This section will apply trial equation scheme to birefringent fibers with two forms of nonlinearity. They are Kerr law and parabolic law. The details are laid out in the next two subsections where the integration algorithm will be successfully implemented.

3.1. Kerr law

The dimensionless form of the coupled nonlinear Schrödinger's equation (NLSE) with group velocity dispersion (GVD) and spatio-temporal dispersion (STD) for Kerr law nonlinearity is given by [1–4,8–10]

$$iq_t + a_1 q_{xx} + b_1 q_{xt} + (c_1|q|^2 + d_1|r|^2)q + i\left\{\alpha_1 q_x + \lambda_1(|q|^2 q)_x + \nu_1(|q|^2)_x q + \theta_1|q|^2 q_x + \gamma_1 q_{xxx}\right\} = 0, \tag{5}$$

$$ir_t + a_2 r_{xx} + b_2 r_{xt} + (c_2|r|^2 + d_2|q|^2)r + i\left\{\alpha_2 r_x + \lambda_2(|r|^2 r)_x + \nu_2(|r|^2)_x r + \theta_2|r|^2 r_x + \gamma_2 r_{xxx}\right\} = 0. \tag{6}$$

In (5) and (6), $q(x, t)$ and $r(x, t)$ are complex valued functions that represents the soliton profiles for the two components in birefringent fibers. For $l = 1, 2$, a_l and b_l represent GVD and STD terms along the two components respectively. It was demonstrated during 2012 that one needs to consider STD in addition to GVD so that the governing equation is well-posed [6,7]. Then, c_l and d_l represents the self-phase modulation and cross-phase modulation terms respectively. In the perturbation terms α_l represents the inter-modal dispersion, λ_l is the self-steepening term, ν_l and θ_l are nonlinear dispersions and finally γ_l is the third order dispersion that must be taken into account whenever GVD or/and STD is/are negligibly small.

In order to solve these equations by the trial equation method, the following solution structure is hypothesized

$$q(x, t) = P_1(\xi)e^{i\phi_1(x,t)}, \tag{7}$$

$$r(x, t) = P_2(\xi)e^{i\phi_2(x,t)}, \tag{8}$$

where the wave variable ξ is given by

$$\xi = k(x - vt). \tag{9}$$