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Optical solitons with Radhakrishnan–Kundu–Lakshmanan equation by extended trial function scheme

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ABSTRACT

This paper extracts optical soliton solutions to Radhakrishnan–Kundu–Lakshmanan model by the aid of extended trial function scheme. Both Kerr and power laws of nonlinearity are considered in this context. Bright, dark and singular soliton solutions are revealed with this integration approach.

1. Introduction

Optical soliton molecule is a treasure trove in the field of nonlinear fiber optics. The entire telecommunication industry is wholly dependent on the dynamics of these molecules in fiber-optic waveguides such as optical fibers, metamaterials and metasurfaces as well as PCF and optical couplers. There are abundant models that formulate the dynamics of such soliton propagation [1–15]. The most visible models are nonlinear Schrödinger's equation, Schrödinger–Hirota equation, Sasa–Satsuma equation, Manakov model, Kundu–Eckhaus equation and many more. These models describe the picture in a variety of situations such as dispersive soliton propagation, differential group delay, polarization-mode dispersion and several others. This paper will study the Radhakrishnan–Kundu–Lakshmanan (RKL) equation that governs soliton propagation dynamics through a polarization-preserving fiber. Our integration technique will be the extended trial function method that will retrieve soliton solutions to the model which are being studied with Kerr and power laws of nonlinearity. Bright, dark and singular soliton solutions are available with the application of the integration scheme to the model of our study in this paper.

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1.1. Mathematical model

The governing equation, in dimensionless form, for the propagation of solitons through an optical fiber is given by the following generalized form of RKL equation [1,8,13]:

$$iq_t + aq_{xx} + bF\left(|q|^2\right)q = i\lambda\left\{F\left(|q|^2\right)q\right\}_x - i\gamma q_{xxx}. \tag{1}$$

In model (1), a and b are real numbers. Here, a represents the coefficient of group velocity dispersion term and b represents the coefficient of nonlinearity. The functional F is the type of nonlinearity that will be considered. On the right-hand side of (1), the coefficient of λ represents the self-steepening term for short pulses [2,8] (typically $\leq$100 fs). The coefficient of third order dispersion term is given by γ. This paper will carry out the integration of the governing equation (1) to derive its soliton and other solutions. This will be possible provided the type of nonlinearity is known. The subsequent two sections details the integration scheme for two different types of nonlinearity.

2. Kerr law

For Kerr law nonlinearity, $F(s) = s$. Thus, the model given by (1) reduces to

$$iq_t + aq_{xx} + b|q|^2 q = i\lambda\left(|q|^2 q\right)_x - i\gamma q_{xxx}. \tag{2}$$

In order to get started, the hypothesis is selected as

$$q(x, t) = P(s)e^{i\phi(x,t)}, \tag{3}$$

where the wave variable s is given by

$$s = x - vt. \tag{4}$$

Here $P(s)$ represents the amplitude portion and v is the speed of the soliton, while the phase portion of the soliton is defined as

$$\phi(x, t) = -\kappa x + \omega t + \theta, \tag{5}$$

where κ is the frequency of the soliton, ω is the wave number, while θ is the phase constant. Finally, $P = P(x, t)$, which represents the pulse shape. Substituting (3)–(5) into (2) and equating the real and imaginary parts respectively gives

$$\left(\omega + a\kappa^2 + \gamma\kappa^3\right)P - (b - \lambda\kappa)P^3 - (a + 3\gamma\kappa)P'' = 0, \tag{6}$$

and

$$\left(v + 2a\kappa + 3\gamma\kappa^2\right)P + \lambda P^3 + \gamma P'' = 0. \tag{7}$$

2.1. Extended trial equation method

This subsection applies the extended trial solution mechanism [3–6,11,12] to handle the RKL equation with Kerr law nonlinearity. To start with the process of extraction of solutions to (6) and (7), the following assumption for the soliton structure is made

$$P = \sum_{i=0}^{\varsigma} \tau_i \Psi^i, \tag{8}$$

where

$$(\Psi')^2 = \mho(\Psi) = \frac{\Phi(\Psi)}{\Upsilon(\Psi)} = \frac{\mu_\sigma \Psi^\sigma + \cdots + \mu_1\Psi + \mu_0}{\chi_\rho \Psi^\rho + \cdots + \chi_1\Psi + \chi_0}. \tag{9}$$

Here $\tau_0, \ldots, \tau_\varsigma$; $\mu_0, \ldots, \mu_\sigma$ and $\chi_0, \ldots, \chi_\rho$ are constants to be determined later. From (8) and (9), terms $(P')^2$ and P'' can be derived as

$$(P')^2 = \frac{\Phi(\Psi)}{\Upsilon(\Psi)}\left(\sum_{i=0}^{\varsigma} i\tau_i\Psi^{i-1}\right)^2, \tag{10}$$

and

$$P'' = \frac{\Phi'(\Psi)\Upsilon(\Psi) - \Phi(\Psi)\Upsilon'(\Psi)}{2\Upsilon^2(\Psi)}\left(\sum_{i=0}^{\varsigma} i\tau_i\Psi^{i-1}\right) + \frac{\Phi(\Psi)}{\Upsilon(\Psi)}\left(\sum_{i=0}^{\varsigma} i(i-1)\tau_i\Psi^{i-2}\right), \tag{11}$$