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Interactions of solitons, dromion-like structures and butterfly-shaped pulses for variable coefficient nonlinear Schrödinger equation

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ABSTRACT

Investigation on interactions of various types of solitons plays an important role in the analysis of various physical mechanisms. In this paper, analytic two- and three-soliton solutions for the variable coefficient nonlinear Schrödinger (vcNLS) equation are obtained with the help of the Hirota method. Through choosing the related parameters of solutions, we present various types of soliton interactions, and analyze the influence of corresponding parameters on soliton interactions. Periodic and oscillatory interactions between parabolic solitons are observed firstly. The method to control the butterfly-shaped pulse interaction is provided. Those results have guiding effect on controlling the interaction of solitons in nonlinear optics and Bose–Einstein condensation.

1. Introduction

Soliton interaction is one of the hot research contents, and has been widely used in ultrafast optics, fluid mechanics, plasma physics, optical communications and Bose–Einstein condensate [1–21]. Theoretical and experimental investigations on soliton interactions have been done by means of the variational method, Hirota method, and split step Fourier transform method [22–33]. Among them, the Hirota method is a simple and direct method to solve partial differential equations and get soliton solutions [5,6]. On the other hand, soliton interactions have been discussed in optical communications. The information in the optical communication system takes solitons as the carrier waves, and the transmission of solitons are affected by their interactions [11]. Thus, it is necessary to study the interactions of various types of solitons.

Under investigated in this paper is the following variable coefficient nonlinear Schrödinger (vcNLS) equation,

$$\frac{\partial A}{\partial z} - i\frac{D(z)}{2}\frac{\partial^2 A}{\partial t^2} + i\rho(z)|A|^2 A = g(z)A. \tag{1}$$

$A(z, t)$ is the temporal envelope of solitons. z is the ordinate, and t is the moving coordinate system of the time. $\rho(z)$ is a Kerr nonlinear coefficient, $g(z)$ is associated with the loss or gain coefficient, and $D(z)$ represents the group-velocity dispersion (GVD) coefficient.

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For Eq. (1), an effective technique for how to control the shape of solitons has been suggested [34–39]. The conditions for the soliton amplification in inhomogeneous systems have been proposed, and the gain of the amplifier on this basis has been discussed [40,41]. Besides, the parallel transmission of solitons has been analyzed when the GVD and nonlinear coefficients are the exponential functions [42]. Moreover, butterfly-shaped pulses have been preliminary explored [43]. Furthermore, dromion-like structures have been reported [44], and the parabolic soliton propagation has been studied [45].

In this paper, two- and three-soliton solutions will be obtained with the Hirota method, and soliton interactions will be analyzed. Various types of solitons, such as parabolic solitons, dromion-like structures and butterfly-shaped pulses, will be presented. The article will unfold as follows: In Section 2, two-soliton solutions for Eq. (1) will be derived. Parallel and parabolic transmission of solitons and dromion-like structures will be obtained and discussed. Then in Section 3, three-soliton solutions for Eq. (1) will be got. Parabolic solitons, dromion-like structures and butterfly-shaped pulses will be demonstrated. Moreover, discussion on those different interactions will be done. The characteristics of soliton interactions and influences of the parameters will be analyzed. In Section 4, the conclusion will be drawn.

2. Analytic two-soliton solutions and discussions

Under the independent variable transformation,

$$A(z,t)=\frac{h(z,t)}{f(z,t)}, \tag{2}$$

where $h(z, t)$ is a complex function, and $f(z, t)$ is a real one, the bilinear forms for Eq. (1) can be obtained as

$$D_z h\cdot f - i\frac{D(z)}{2}D_t^2 h\cdot f - g(z)h\cdot f = 0,$$
$$D(z)D_t^2 f\cdot f + 2\rho(z)hh^* = 0.$$

Here, * represents the complex conjugate. Moreover, D_z and D_t are bilinear operators which can be defined by [46]

$$D_z^m D_t^n(G\cdot F)=\left(\frac{\partial}{\partial z}-\frac{\partial}{\partial z'}\right)^m\left(\frac{\partial}{\partial t}-\frac{\partial}{\partial t'}\right)^n G(z,t)F(z',t')|_{z'=z,t'=t}.$$

The bilinear forms can be solved by the power series expansions of $h(z, t)$ and $f(z, t)$ as follows,

$$h(z,t)=\varepsilon h_1(z,t)+\varepsilon^3 h_3(z,t)+\varepsilon^5 h_5(z,t)+\cdots,$$
$$f(z,t)=1+\varepsilon^2 f_2(z,t)+\varepsilon^4 f_4(z,t)+\varepsilon^6 f_6(z,t)+\cdots,$$

where ε is a formal expansion parameter.

To obtain two-soliton solutions, we assume that

$$h(z,t)=\varepsilon h_1(z,t)+\varepsilon^3 h_3(z,t),$$
$$f(z,t)=1+\varepsilon^2 f_2(z,t)+\varepsilon^4 f_4(z,t),$$

where

$$h_1(z,t)=e^{Q_1(z,t)}+e^{Q_2(z,t)}, f_4(z,t)=m_5(z)e^{Q_1(z,t)+Q_2(z,t)+Q_1^*(z,t)+Q_2^*(z,t)},$$
$$h_3(z,t)=n_1(z)e^{Q_1(z,t)+Q_2(z,t)+Q_1^*(z,t)}+n_2(z)e^{Q_1(z,t)+Q_2(z,t)+Q_2^*(z,t)},$$
$$f_2(z,t)=m_1(z)e^{Q_1(z,t)+Q_1^*(z,t)}+m_2(z)e^{Q_2(z,t)+Q_1^*(z,t)}+m_3(z)e^{Q_1(z,t)+Q_2^*(z,t)}$$
$$+m_4(z)e^{Q_2(z,t)+Q_2^*(z,t)}.$$

What is more,

$$Q_1(z,t)=k_{11}(z)+ik_{12}(z)+(w_{11}+iw_{12})t+\theta_{11}+i\theta_{12},$$
$$Q_2(z,t)=k_{21}(z)+ik_{22}(z)+(w_{21}+iw_{22})t+\theta_{21}+i\theta_{22}.$$