SYNCHRONIZATION AND CONTROL IN ENSEMBLES OF PERIODIC AND CHAOTIC NEURONAL ELEMENTS WITH TIME DEPENDENT COUPLING

Vladimir N. Belykh and Evgeniya V. Pankratova *

* *Mathematics Department, Volga State Academy,*
5, Nesterov street, Nizhny Novgorod, 603600, Russia.
belykh@unn.ac.ru, pankratova@aqua.sci-nnov.ru

Abstract: The study of complete synchronization in networks of periodic and chaotic neurodynamical elements with different coupling configurations is performed. Using the connection graph stability method we obtain the sufficient conditions for achievement of synchronous behavior of all elements involved in these ensembles. The theoretical predictions we compare with the numerical results obtained for the networks composed of the classical Hodgkin-Huxley neuronal elements. The problem how to control the synchronization of networks growing in time is discussed.

Keywords: Neural networks, synchronization, stability criteria, chaotic behavior

1. INTRODUCTION

The stability of a synchronous state in large ensembles of coupled oscillators is one of the most intensively studied problem arising in different fields of science. This topic is of significant interest in the context of electronic circuits, chemical and biological systems, and secure communication (for particular examples see (Pikovsky *et al.*, 2001)).

In the present work we study the complete synchronization in the context of neural networks. The dynamics of individual element of the network is described by the classical Hodgkin-Huxley equations (Hodgkin and Huxley, 1952). Several types of possible topology for the network are examined. Among the objectives of the study of such networks is to get a better understanding of basic mechanisms of sensory processing, motor control, memory and higher information-processing functions of the brain. From experimental works of Swadlow (1992) on mammalian neocortex it is known that the delays on the synaptic connections could be small enough. In this case a synfire activity of the neurons in a network should be considered (Izhikevich, 2006). Only such type of activity can provide further transmission of information along the network. The point is that, the synchronously generated spikes arrive to the target at the same time, thereby evoking potent postsynaptic responses. If the neurons fire asynchronously their spikes arrive to the postsynaptic target at different times evoking only weak or no response. In this context, the stability of the synfire activity is of great importance.

In order to determine the stability, various criteria can be used (Pecora and Carroll, 1998; Pogromsky and Nijmeijer, 2001; Wu and Chua, 1996). In this work the results of theoretical prediction obtained within the framework of recently developed con-

[1] This work was supported in part by the Russian Foundation for Basic Research (grants No. 05-01-00509 and No. 07-02-01404) and NWO-RFBR (grant No. 047-017-018). E.V.P. also acknowledges the support of the Dynasty Foundation.

nection graph stability method (Belykh *et al.*, 2004) are presented. These results are compared with the data of numerical calculations. Some aspects of the synchronization in multilayered neuronal networks that are suggestive of sensory-motor systems are also touched upon.

2. COMPLETE SYNCHRONIZATION: THE STATE OF THE PROBLEM

Let us consider a network of n coupled identical oscillators:

$$\dot{\mathrm{x}}_i = F(\mathrm{x}_i) + \sum_{j=1}^{n} \varepsilon_{ij}(t)\mathcal{P}\mathrm{x}_j, \quad i = 1, ..., n \quad (1)$$

Here $\mathrm{x}_i = (x_i^1, x_i^2, ..., x_i^d)$ is the d-vector containing the coordinates of the i-th oscillator, $F(\mathrm{x}_i)$ is a nonlinear vector function defining the dynamics of the individual element. The non-zero elements of the $(d \times d)$ matrix $\mathcal{P} = diag(p_1, p_2, ..., p_d)$, where $p_h = 1$ for $h = 1, 2, ..., s$ and $p_h = 0$ for $h = s+1, ..., d$ determine which variables couple the individual systems.

The matrix $\mathcal{G} = \{\varepsilon_{ij}(t)\}$ is an $(n \times n)$ symmetric matrix with non-negative off-diagonal elements. The diagonal elements of the connectivity matrix are chosen from a necessary condition for the existence of the synchronous solution of the system (1), namely, the invariance of hyperplane $M = \{\mathrm{x}_1(t) = \mathrm{x}_2(t) = ... = \mathrm{x}_n(t)\}$. This means that diagonal elements of the matrix $\mathcal{G}$ are assumed to be equal $\varepsilon_{ii} = -\sum_{j=1;j\neq i}^{n} \varepsilon_{ij}$, $i = 1, 2, ..., n$. The global asymptotical stability of the invariant manifold M corresponds to the completely synchronous state of the network. In this case any trajectory of the system (1) unrestrictedly converges to any attractor on M.

The connectivity matrix $\mathcal{G}$ defines a graph with n vertices and m edges. The number of edges m equals the number of non-zero above diagonal elements ε_{ij}. The i-th vertex of the graph corresponds to the i-th oscillator of the network. Therefore, if l-th and k-th oscillators of the ensemble are coupled, i.e. $\varepsilon_{lk} = \varepsilon_{kl} > 0$, then the corresponding graph has the edge linking l-th and k-th vertices. Between these vertices there is a path with the unit length P_{lk}. For the general case the length of the path $z(P_{ij})$ equals to the number of edges involved in P_{ij}, that links i-th and j-th vertices in accordance with the connectivity matrix $\mathcal{G}$.

The main statement of the connection graph stability method is that for the definite conditions (see (Belykh *et al.*, 2004)) synchronization manifold $M = \{\mathrm{x}_1(t) = \mathrm{x}_2(t) = ... = \mathrm{x}_n(t)\}$ is globally asymptotically stable if the following inequality holds:

$$\varepsilon_k(t) > \varepsilon_k^* = \frac{a}{n} b_k(n, m) \quad (2)$$

where $b_k(n,m) = \sum_{j>i;k\in P_{ij}}^{n} z(P_{ij})$ is the sum of the lengths of all chosen paths P_{ij} which pass through a given edge k that belongs to the coupling configuration. The parameter a is a constant related to the dynamical properties of the individual dynamical systems.

In general, the dynamics of the elements in networks can be described by an arbitrary model. In computational neuroscience there are a lot of mathematical models illustrating the richness and complexity of spiking behavior of individual neurons. These models are defined at a different level of abstraction and trying to simulate different aspects of neural systems. The choice of a certain model depends on the type of the problem. This could be, for example, some conductance-based models such as Morris-Lecar describing oscillations in barnacle giant muscle fiber, or Wilson model for cortical neurons, etc. This could be some phenomenological neuronal models such as FitzHugh-Rinzel or Hindmarsh-Rose model, etc. Therefore, in the following theoretical approach the synchronization threshold of the form

$$\widetilde{\varepsilon_k^*} = \frac{\varepsilon_k^*}{a} = \frac{b_k(n,m)}{n}, \quad (3)$$

will be considered. According to (3), the variety of the sums $b_k(n,m)$ gives the variety of synchronization thresholds $\widetilde{\varepsilon_k^*}$, that are sufficient to achieve globally stable synchronization in system (1).

3. TWO STAR-COUPLED NETWORKS CONNECTED BY THE CHAIN

In this section let us consider a network composed of n elements, whose topology is illustrated in Fig. 1. From neurophysiological point of view this type of the structure corresponds to a couple of diffusively connected pacemaker neuronal cells. For convenience, we introduce the following notations: m_c is the number of elements in the chain linking the central nodes of the stars; m_{st}^l and m_{st}^s are the numbers of elements for the most loaded star and the star with low concentration of load, respectively. Thus, the total number of cells in the network is $n = m_{st}^l + m_{st}^s + m_c$.

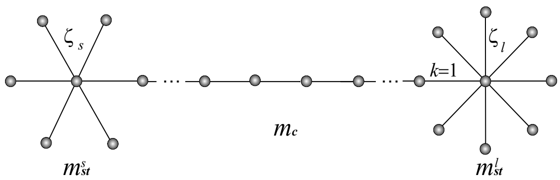

Fig. 1. The coupling structure of an ensemble with two stars connected by the chain.