Original research article

Reduced density matrix identification of open quantum system

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A R T I C L E   I N F O

A B S T R A C T

Identification of performance parameter of a multi-level open quantum system is a central problem in a growing number of applications of information engineering. Based on the quantum tomography, this paper aims at finding a method for identifying reduced density matrix of open quantum system. State identification of open quantum system approaches is presented for two-level, three-level and N-level systems, respectively. The results show that not only the reduced density matrix can be reestablished, but also the quantum state information can be received in quantum tomography experiment.

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1. Introduction

Recently, much effort has been put into the design and realization of large scale quantum devices operating. This has been spurred by the possibilities offered by quantum communication and information processing, from secure transmission, simulation of quantum dynamics, and the solution of currently intractable mathematical problems. Many different physical systems have been proposed as basic architectures upon which to construct quantum devices, ranging from atoms, ions, photons, quantum dots and superconductors [1–3]. For large scale commercial applications, it is likely that this will involve scalable engineered and constructed devices with tailored dynamics requiring precision control. Then characterization and control of quantum systems is a key problem in quantum computation and control [4–8].

Quantum state as a unit of quantum information is the main research object in quantum computation and control [9–11]. Since quantum states follow the laws of quantum mechanics, quantum non-cloning theorem and uncertainty relationship result in "quantum collapse" phenomenon when quantum states are measured, which makes information of quantum system acquisition much difficult.

Indeed, the system real state cannot be measured directly. We can only measure the collapse probability of quantum system in a projection direction, which is fundamentally different from the features of macro system. Therefore measure can only estimate the real state in the quantum system (quantum state reconstruction). At present, the quantum state estimation is divided into three main aspects: the estimation of the initial state of the finite dimensional quantum system [12]; the evolution of a series and the information based on an observable quantity state estimation [13]; state estimation based on historical records of continuous measurement of individual quantum systems [14]. The last aspect, also known as the estimation of quantum processes.

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Quantum tomography is a standard technique for the estimation of the quantum state. The term, quantum tomography, comes from medical X-ray computer-aided tomography (CT), and it is a statistical measurement method [15]. According to quantum mechanics we know that a quantum system density matrix contains all the system information, so the system information is totally obtained if the system density matrix can be acquired. Quantum tomography can reconstruct density matrix by measuring a large number of unknown quantum state in the same status to get quantum state information. The results show that quantum tomography is an effective and feasible method to obtain quantum state information.

In Ref. [16], the authors studied the minimal informational complete measurements for pure states, and discussed the pure state reconstruction based on measurement outcomes. A paper published in 2011 initiated another approach to quantum tomography which is based on weak measurement. The paper revealed that the wave function of a pure state can be measured in a direct way [17]. Further papers proved that this approach can be generalized also for mixed state identification [18]. Practical quantum systems are open quantum systems because of the interaction with the external environment, which makes the system model more complicated. Some celebrated results have been achieved on the estimation of unknown dynamical parameters of open quantum systems [19].

Different from classical systems, a closed or open quantum system described by the quantum Liouville equation is a complex-valued dynamics due to the physical properties of quantum states. To exploit the well-developed results from classical computer-aided tomography and control theory into quantum control, one of the inspiring strategies is to derive a real-valued dynamics to replace the complex-valued quantum Liouville equation.

In this paper, we discuss quantum state reconstruction of open quantum systems based on reduced density matrices. We will show that a quantum state is characterized of a set of real value parameters and thus can be uniquely determined by a series of measurements. Based on the discussion on the quantum state identification, we will propose a strategy to obtain the real-valued dynamics. Specially, we will deduce the dynamics for n-level system.

The rest of this chapter is organized as follows. In Section 2, the quantum state identification strategy is presented. In Sections 3, the real-valued equations are deduced for two- and three-level systems, respectively. In Section 4, the results are extended to n-level systems. Section 5 is for our conclusions.

2. Quantum dynamics

In quantum mechanics, the state of quantum system can be described by many ways. A quantum system with a state vector $|\psi\rangle$ is called a pure state, which can be described by the wave function that evolves according to Schrodinger equation. However, it is also possible for a system to be in a statistical ensemble of different state vectors. Mixed states arise in situations where the experimenter does not know which particular states are being manipulated. Examples include a system in thermal equilibrium or a system with an uncertain or randomly varying preparation history (so one does not know which pure state the system is in). Also, if a quantum system has two or more subsystems that are entangled, then each subsystem must be treated as a mixed state even if the complete system is in a pure state. The density matrix ρ is especially useful for mixed states, because any state, pure or mixed, can be characterized of a single density matrix.

The density matrix is a representation of a linear operator called the density operator. The density matrix is obtained from the density operator by choice of basis in the underlying space. In practice, the terms density matrix and density operator are often used interchangeably. The density matrix or density operator ρ is self-adjoint (or Hermitian), positive semi-definite, of trace one, and may be infinite-dimensional.

For a finite-dimensional function space, the most general density operator is of the form

$$\rho = \sum_j p_j |\psi_j\rangle \langle \psi_j|, \tag{1}$$

where the coefficients p_j are non-negative and add up to one. This represents a statistical mixture of pure states. If the given system is closed, then one can think of a mixed state as representing a single system with an uncertain preparation history, as explicitly detailed above; or we can regard the mixed state as representing an ensemble of systems, i.e. a large number of copies of the system, where p_j is the proportion of the ensemble being in the state $|\psi_j\rangle$.

It can not only denote pure state but also denote mixed state, especially can conveniently extend to infinite dimensional physical space. Therefore, the density operator ρ is adopted to represent the state of system. Just as the Schrödinger equation describes how pure states evolve in time, the Liouville equation describes how a density operator evolves in time. In the Hilbert space H, the dynamical evolution of the system state satisfies the quantum Liouville equation [20]

$$\frac{d\rho}{dt} = -i[H_0 + H_C, \rho] + D[\sigma]\rho, \tag{2}$$

where the brackets denote a commutator $[A, B] = AB - BA$. $D[\sigma]\rho$ is the super-operator, which depends on the interaction between system and surrounding environments.

Let F be an observable of the system, the expectation value of the measurement can be calculated

$$\bar{F} = tr(\rho F). \tag{3}$$

Different from many classical systems, the evolution of a close or open quantum system is described by the Liouville equation, which is a complex-valued equation due to the physical properties of quantum states. It is emphasized that, for