OPTIMAL CONTROL OF PERIODIC MOTIONS OF VIBRATION-DRIVEN SYSTEMS

Nikolai N. Bolotnik [*,1] **Tatiana Yu. Figurina** [*,1]

[*] *Institute for Problems in Mechanics of the Russian Academy of Sciences, 101-1 Prospect Vernadskogo, Moscow 119526, Russia*
bolotnik@ipmnet.ru; figur@ipmnet.ru

Abstract: An optimal control problem is solved for a rigid body that moves along a straight line on a rough horizontal plane due to the motion of two internal masses. One of the masses moves horizontally parallel to the line of motion of the system's main body and the other mass moves vertically. Such a mechanical system models a vibration-driven robot able to move in a resistive medium without special propelling devices (wheels, legs or caterpillars). A periodic motion of the internal masses is constructed to ensure a velocity-periodic motion of the main body with a maximum average velocity, provided that the period is fixed and the accelerations of the internal masses relative to the main body lie within prescribed limits. This statement does not constrain the amplitude of vibrations of the internal masses. Based on the solution of the problem, a suboptimal control that takes this constraint into account is constructed.

Keywords: Vibration-driven Systems, Optimal Control, Robotics

1. INTRODUCTION

A rigid body with internal masses that perform periodic motions can move progressively in a resistive medium with nonzero average velocity. This phenomenon can be used as a basis for the design of new-type mobile systems able to move without special propelling devices (wheels, legs, caterpillars or screws) due to direct interaction of the body with the environment. Such systems have a number of advantages over systems based on the conventional principles of motion. They are simple in design, do not require gear trains to transmit motion from the motor to the propellers, and their body can be made hermetic and smooth, without any protruding components. The said features make this principle of motion prospective for being used in capsule-type microrobots designed for motion in strongly restricted space (e.g., inside narrow tubes) and in vulnerable media, for example, inside a human body for delivering a drug or a diagnostic sensor to an affected organ. Automatic transport systems moving due to periodic motion of internal masses are sometimes referred to as vibration-driven robots. Issues of control and optimization of motion of systems with internal movable masses have been studied by Chernousko (2002, 2005, 2006) and Figurina (2007). The dynamics and design of vibration-driven robots have been considered by Gradetsky *et al.* (2003), Li *et al.* (2005), Chernousko *et al.* (2005), Bolotnik *et al.* (2006), and Vartholomeos and Papadopoulos (2006).

In the present paper, a vibration-driven system consisting of a main body and two internal masses

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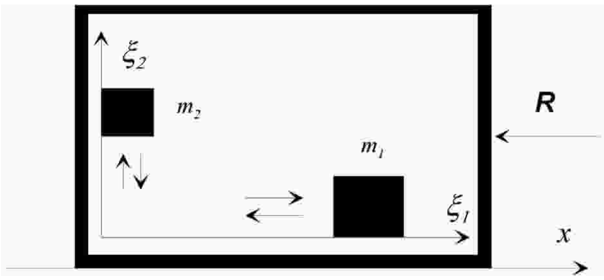

Fig. 1. Schematic of the system

is considered. The main body is based on a rough horizontal plane and can move along a straight line over the plane. There is dry (Coulomb's) friction acting between the body and the plane. One of the internal masses moves horizontally along a straight line parallel to the line of motion of the body, while the other mass moves vertically. The motion of the internal masses is controlled by forces acting between the masses and the body. Therefore, the control of the horizontal motion of the internal mass enables one to control the magnitude and direction of the friction force applied to the body, which provides the progressive motion of the entire system. The control of the vertical motion of the internal mass ensures an additional possibility of control of the dry friction magnitude due to the change of the normal pressure force exerted on the body by the supporting surface.

Periodic modes of motion of the internal masses are constructed to provide a velocity-periodic progressive motion of the main body with a maximum average velocity. The average velocity of the steady-state motion of the body is a basic operating characteristic of vibration-driven robots, and the maximization of this velocity is an important task for planning motions of such systems.

2. DESCRIPTION OF THE MODEL

Consider a rigid body of mass m_0 that is able to move along a straight line on a rigid rough plane. Inside this body, there are two movable internal point masses m_1 and m_2. Mass m_1 moves horizontally along a line parallel to the line of motion of the body and mass m_2 moves vertically. The system described is shown in Fig. 1.

The system is controlled by moving the internal masses relative to the body due to internal forces acting between the masses and the body. Let x be the displacement of the body relative to a fixed (inertial) reference frame, ξ_1 the horizontal displacement of mass m_1 relative to the body, ξ_2 the vertical displacement of mass m_2 relative to the body, and R the friction force exerted on the body by the supporting plane. Let the friction be dry friction modeled by Coulomb's law. We assume that the x and ξ_1 axes are co-directed and that the ξ_2 axis points vertically upward. Then the motion of the body is governed by the relations

$$M\ddot{x} + m_1\ddot{\xi}_1 = R, \quad M = m_0 + m_1 + m_2, \qquad (1)$$

$$R = \begin{cases} -kN\mathrm{sign}\dot{x}, \text{ if } \dot{x} \neq 0, \\ m_1\ddot{\xi}_1, \text{ if } \dot{x} = 0 \text{ and } |m_1\ddot{\xi}_1| \leq kN, \\ kN\mathrm{sign}(m_1\ddot{\xi}_1), \\ \text{if } \dot{x} = 0 \text{ and } |m_1\ddot{\xi}_1| > kN, \end{cases} \qquad (2)$$

$$N = Mg + m_2\ddot{\xi}_2, \qquad (3)$$

where k is the coefficient of friction between the supporting plane and the body, g is the acceleration due to gravity, and N is the normal pressure force applied to the body by the plane. Since the plane resists the penetration of the body but does not resist the separation, the quantity N must be nonnegative. Therefore, in accordance with (3), the contact of the body with the plane implies the inequality

$$Mg + m_2\ddot{\xi}_2 \geq 0. \qquad (4)$$

3. STATEMENT OF THE OPTIMAL CONTROL PROBLEM

Periodic motions of the internal masses will be constructed to provide a velocity-periodic motion of the body with a maximum average velocity for a prescribed period T. Due to the periodicity, it suffices to construct the desired motion on the interval $0 \leq t \leq T$. Assume without loss of generality that

$$\xi_1(0) = 0, \quad \xi_2(0) = 0, \quad x(0) = 0. \qquad (5)$$

These initial conditions are ensured by an appropriate choice of origin for the respective coordinates. The periodicity of the functions $\xi_1(t)$, $\xi_2(t)$, and $\dot{x}(t)$ implies the relations

$$\begin{gathered} \dot{x}(0) = \dot{x}(T) \\ \xi_i(0) = \xi_i(T) = 0, \quad \dot{\xi}_i(0) = \dot{\xi}_i(T), \quad i = 1,\ 2. \end{gathered} \qquad (6)$$

The accelerations of the internal masses relative to the body will be taken as the control variables subject to the constraints

$$|\ddot{\xi}_1(t)| \leq U_1, \quad -U_2^- \leq \ddot{\xi}_2(t) \leq U_2, \qquad (7)$$

$$\int_0^T \ddot{\xi}_i(t)dt = 0, \quad i = 1,\ 2, \qquad (8)$$

where

$$U_2^- = \min\left\{U_2, \frac{Mg}{m_2}\right\}. \qquad (9)$$