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# Optical solitons for the Kundu–Eckhaus equation with time dependent coefficient

Mustafa Inc[a,*], Dumitru Baleanu[b,c]

[a] Fírat University, Science Faculty, Department of Mathematics, 23119 Elazığ, Turkey
[b] Department of Mathematics, Cankaya University, Ankara, Turkey
[c] Institute of Space Sciences, Magurele, Romania

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ABSTRACT

The first integral method (FIM) is applied to get the different type optical solitons of Kundu–Eckhaus equation (KE). A class of optical solitons of this equation is presented, and some of which are acquired for the first time. Constraint conditions guarantees existence of these solitons. It is illustrated that FIM is very effective method to reach the various type of the soliton solutions.



## 1. Introduction

Nonlinear partial differential equations are generally used for the expression of mathematical modeling of scientific systems. This shows that it is relevant to acquire general solutions of the associated nonlinear equations. Therefore, the general solutions of these equations supply lots of ideas and logic about the characters and the structures of the equations for different authors. In literature, there are many methods for obtaining soliton solutions of NPDEs [1–7].

Feng [7] initially proposed the FIM to the literature by solving Burgers–KdV equation. The FIM has been successfully use to solve NPDEs and some fractional differential equations that are new types of equations. Recently, lots of studies have been made by using the FIM [8–22].

The article is organized as follows: In Section 2, the FIM has been presented. In Section 3, this method is applied into the KE equation that was constructed by Kundu and Eckhaus [23–27], as a linearizable form of the nonlinear Schrödinger equation. We give final remarks in the last section.

## 2. Description of the first integral method

The FIM consists of the following steps:

**Step 1.** Considering a NPDE of the form:

$$W(h, h_t, h_x, h_{xt}, h_{tt}, h_{xx}, \ldots) = 0. \tag{1}$$

* Corresponding author.
*E-mail addresses:* minc@firat.edu.tr (M. Inc), dumitru@cankaya.edu.tr (D. Baleanu).

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So that Eq. (1) can be transformed to an ODE below:

$$L(Q, Q', Q'', Q''', \ldots) = 0 \tag{2}$$

such that $\xi = x \mp ct$ and $Q' = \partial Q(\xi)/\partial \xi$.

**Step 2.** Suppose that from Eq. (2) we have

$$h(x, t) = h(\xi). \tag{3}$$

**Step 3.** A new independent variable is introduced as

$$H(\xi) = h(\xi), \quad G(\xi) = \partial h(\xi)/\partial \xi \tag{4}$$

which presents a new system of ODEs

$$\begin{aligned} &\partial H(\xi)/\partial \xi = G(\xi) \\ &\partial F(\xi)/\partial \xi = P(H(\xi), G(\xi)) \end{aligned} \tag{5}$$

**Step 4.** Based on the qualitative theory of differential equations [28], if reaching the integral for system (5) is possible, then the solutions of system (5) can immediately be obtained. In a situation of some independent plane system, there is no general theory telling us how to find its first integrals in a systematic way. The division theorem (DT) [29] provided us with an idea on how to reach the first integrals.

## 3. Application

In this section, we consider the KE equation with time dependent coefficient as

$$ih_t + h_{xx} + \left(\alpha|h|^{2n} + \beta|h|^{4n}\right) h + i\gamma\left(|h|^{2n}\right)_x h = 0, \tag{6}$$

where $\alpha = \alpha(t)$, $\beta = \beta(t)$ and $\gamma = \gamma(t)$.

Eq. (6) turns to the following ODE system in real and imaginary parts by using the wave variable

$$h = H\left(\xi\right) e^{i[w(\xi) - vt]}, \quad \xi = \kappa x - \mu t,$$

$$\kappa^2 H_{\xi\xi} + vH + \alpha H^{2n+1} + \beta H^{4n+1} + \mu H w_\xi - \kappa^2 H w_\xi^2 = 0, \tag{7}$$

$$\mu H_\xi - 2\gamma\kappa k H^n H_\xi - 2\kappa^2 H_\xi w_\xi - \kappa^2 H w_{\xi\xi} = 0. \tag{8}$$

Then, by using (8) we have

$$w_\xi = \frac{\mu}{2\kappa^2} - \frac{\gamma n}{\kappa(n+1)} H^n. \tag{9}$$

By substituting (9) into (7)

$$\begin{aligned} &\left(n^2 + 1\right)\left(\mu^2 + 4\kappa^2 v\right) H + 4\kappa^2\left[\alpha\left(n^2 + 1\right) - \gamma^2 n^2\right] H^{2n+1} \\ &\quad + 4\beta\kappa^2(n+1)^2 H^{4n+1} + 4\kappa^2(n+1)^2 H_{\xi\xi} = 0, \end{aligned} \tag{10}$$

then with the transformation $H_\xi = G$, we have

$$\begin{aligned} &H_\xi = G, \\ &G_\xi = -\frac{\left(n^2+1\right)\left(\mu^2 + 4\kappa^2 v\right)}{4\kappa^2(n+1)^2} H - \frac{4\kappa^2\left[\alpha\left(n^2+1\right) - \gamma^2 n^2\right]}{4\kappa^2(n+1)^2} H^{2n+1} - \beta H^{4n+1}. \end{aligned} \tag{11}$$

In accordance with the FIM, it is supposed that $H(\xi)$and $G(\xi)$ are non-trivial solutions of Eq. (11) and $F(H, G) = \sum_{i=0}^{r} a_i(H) G^i$ is an irreducible function in the domain $C[H, G]$ such that

$$F(H(\xi), G(\xi)) = \sum_{i=0}^{r} a_i(H) G^i = 0, \tag{12}$$

where $a_i(H)$, $(i = 0, 1, 2, \ldots, r)$ are polynomials of $H$ and $a_r(H) \neq 0$. Eq. (12) is the first integral for system (11), owing to the DT, there exists $g(H) + h(H)G$ in $C[H, G]$ as:

$$\begin{aligned} dF/d\xi &= \frac{dF}{dH}\frac{dH}{d\xi} + \frac{dF}{dG}\frac{dG}{d\xi} \\ &= [g(H) + h(H)G]\sum_{i=0}^{r} a_i(H) G^i. \end{aligned} \tag{13}$$