Optik 158 (2018) 112–117

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Original research article

Conversions and interactions of the nonlinear waves in a generalized higher-order nonlinear Schrödinger equation

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ARTICLE INFO

Article history:
Received 31 October 2017
Received in revised form
23 November 2017
Accepted 23 November 2017

Keywords:
Breather-to-soliton dynamics
Antidark soliton
Multi-peak soliton
Periodic wave

ABSTRACT

In this paper, we investigate a generalized higher-order nonlinear Schrödinger (NLS) equation with two free parameters, including the third-order and fourth-order dispersion with matching higher-order nonlinear effects. The results show that the breather solution can be converted into some types of localized and periodic waves under specified parameter conditions. Coupled with rich graphical examples, the coexistence and interaction of different nonlinear structures are displayed. Further, we demonstrate the explicit relation on the parallel propagation of the second-order breather solution.

1. Introduction

Several types of exact solutions have been investigated in many nonlinear evolution equations [1–16]. Solitons which arise due to the balance of the dispersive and nonlinear terms are a class of stable waves [1–4,17–19]. Under the continuous wave background the breather solution is often established. Breathers can be seen as the localized oscillating solitons in which the energy distribution is localized and periodic. According to the propagation characteristics, they are divided into the Kuznetsov–Ma breather (periodic in time but localized in space) and the Akhmediev breather (periodic in space but localized in time) [5,6]. There exists another kind of the localized solution, the Peregrine soliton (PS), which can well describe the rogue waves (RWs) [7,16]. As the limiting case of the breather solution, it is localized both in spatial and time. Recent studies have shown that the higher-order terms can lead to the conversion of nonlinear localized and periodic waves on the continuous wave background [20–23]. The dynamic breather-to-soliton and RWs-to-soliton conversions have been established in some nonlinear systems [24–26].

In this paper, we focus on a generalized higher-order nonlinear Schrödinger (NLS) equation in the operator form

$$i\psi_x + S[\psi(x,t)] - i\alpha H[\psi(x,t)] + \gamma P[\psi(x,t)] = 0, \tag{1.1}$$

where

$$S[\psi(x,t)] = \frac{1}{2}\psi_{tt} + \psi|\psi|^2, \quad H[\psi(x,t)] = \psi_{ttt} + 6\psi_t|\psi|^2,$$
$$P[\psi(x,t)] = \psi_{tttt} + 6\psi|\psi|^4 + 4\psi|\psi_t^*|^2 + 8\psi_{tt}|\psi|^2 + 6\psi_t^2\psi^* + 2\psi^2\psi_{tt}^*.$$

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https://doi.org/10.1016/j.ijleo.2017.11.195

S, H and P denote the nonlinear Schrödinger operator, the Hirota operator, and the Lakshmanan–Porsezian–Daniel (LPD) operator respectively. Eq. (1.1) has two independent coefficients: α and γ, in which the third-order and fourth-order linear dispersion and other nonlinear terms such as Kerr effects, cubic quintic nonlinearity, self-steepening and self-frequency shift are taken into consideration. When $\gamma = 0$, without considering the LPD operator, Eq. (1.1) becomes the Hirota equation [22]. If $\alpha = 0$, Eq. (1.1) is known as the LPD equation modeling a general class of the one-dimensional continuum Heisenberg spin chain [23].

The integrability, Lax pair, and lower-orders soliton solutions of Eq. (1.1) have been demonstrated [27] and the existence of N-soliton by the bilinear method has been investigated in [28]. Furthermore, the Kuznetsov–Ma breather and Akhmediev breather on the continuous wave background has presented [29]. In this paper, we display a breather solution expression with two free parameters on the plane-wave background and derive some new types of nonlinear localized and periodic solutions, such as multi-peak soliton, antidark soliton, W-shaped soliton and periodic wave. Finally, we provide some examples of breathers into solitons and analyze the interaction behaviors of these nonlinear waves.

2. Different types of nonlinear waves

For Eq. (1.1), we can use the Darboux transformation (DT) technique to generate the breather solution on the plane wave background. Substituting the seed solution $\psi^{[0]} = k\exp[i(Gx + Ft)]$ by the Lax pair given in [27], we obtain the first order linear functions r_1 and s_1 in the form

$$r_1 = 2ie^{-(i/2)(1+6\gamma)x}\sin[B], \tag{2.1a}$$

$$s_1 = 2e^{(i/2)(1+6\gamma)x}\cos[C]. \tag{2.1b}$$

From the onefold DT, the first-order breather solution is

$$\psi^{[1]} = \left(1 + 2b\frac{N_r + iN_i}{D_1}\right)e^{ix(1+6\gamma)}, \tag{2.2}$$

where

$$\kappa = \kappa_r + i\kappa_i = \sqrt{1+\lambda^2} = \sqrt{1+(a+ib)^2}, \quad A = A_r + iA_i,$$

$$\chi = \chi_r + i\chi_i = \arccos[\kappa], \quad B = A - \chi/2, \quad C = A + \chi/2,$$

$$N_r = \cosh[2A_i]\sin[\chi_r] - \sin[2A_r]\cosh[\chi_i],$$

$$N_i = \sinh[2A_i]\cos[\chi_r] - \cos[2A_r]\sinh[\chi_i],$$

$$D = \cosh[2A_i]\cosh[\chi_i] - \sin[2A_r]\sin[\chi_r].$$

λ is a spectral parameter, and

$$A_r = t\kappa_r + zV_r, \qquad A_i = t\kappa_i + zV_i,$$

$$V_r = d_r\kappa_r - d_i\kappa_i, \qquad V_i = d_r\kappa_r + d_i\kappa_i,$$

$$d_r = a + \alpha\eta_1 + \gamma\eta_2, \quad d_i = b - \alpha\eta_3 + \gamma\eta_4,$$

with $\eta_1 = 2(1 - 2a^2 + 2b^2)$, $\eta_2 = 2a(\eta_1 + 8b^2)$, $\eta_3 = 8ab$ and $\eta_4 = 2b(\eta_1 - 8a^2)$. To simplify we fix the values of the amplitude k and the frequency F of the background in the above expression. Solution (2.2) depends on four independent parameters: the coefficients of the higher-orders effects α, γ, and the real parameters a, b. By adjusting their values to meet certain relations, we can present the breather-to-soliton conversions and derive different types of wave structures in solution (2.2). Since both N and D are composed of the trigonometric function and the hyperbolic function. Solution (2.2) characterizes an unified structure combined a soliton and a periodic wave with the propagation velocities V_r/κ_r and V_i/κ_i respectively. Based on the velocity difference, some new types of nonlinear wave structures can be clearly demonstrated in the following two cases:

Case 1 – $V_r/\kappa_r \neq V_i/\kappa_i$. Solution (2.2) shows an one-breather structure on the plane-wave background. Further taking $a = -F/2$, we have the time-periodic Kuznetsov–Ma breather with $|k| < |b|$ and the space-periodic Akhmediev breather with $|k| > |b|$. When $|k| \to |b|$, the period tends to infinity. Solution (2.2) describes the Peregrine soliton(PS) which is localized both in time and space.

Case 2 – $V_r/\kappa_r = V_i/\kappa_i$. In this case, $d_j = 0$. The breather can be converted to other nonlinear waves in solution (2.2). The exact conversion relation can be obtained

$$4(1 - 6a^2 + 2b^2)\gamma = 8a\alpha - 1. \tag{2.3}$$

The higher-order coefficients α and γ cannot be zero simultaneously in this conversion. Based on parametric analysis, several types of localized and periodic structures are displayed in Figs. 1–7. If $a \neq -F/2$, then $\kappa_r \neq 0, \kappa_i \neq 0$. Solution (2.2) presents the multi-peak soliton, which is a localized one-soliton with some oscillation or main peaks [23,30]. This structure can