Optik 158 (2018) 177–184

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Original research article

Rogue waves for the optical fiber system with variable coefficients

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ARTICLE INFO

Article history:
Received 19 November 2017
Accepted 22 December 2017

Keywords:
Nonlinear optical fiber system
Generalized variable coefficient
Schrödinger equation
Similarity transformation
Rogue wave

ABSTRACT

The rogue waves are investigated in the generalized nonlinear optical fiber system, which is modelled as a generalized variable coefficient Schrödinger equation distributed with two group velocity dispersion functions and a nonlinearity function. The rogue wave solutions are constructed via similarity transformation. Furthermore, the first- and second-order rogue waves are excited via choosing group velocity dispersion coefficients as specific function. The rogue waves exhibit abundant characteristics in shape, amplitude, peak number and stretch, while the rogue waves also are controllable through the system parameters. The results reveal partially the formation mechanism of the optical rogue waves under variable group velocity dispersion.

1. Introduction

The optical rogue waves (RWs) were firstly introduced in Refs. [1,2], which reported that especially high wave peaks were observed in extremely broadband radiation generated from a narrow-band input in a micro-structured and optical nanoscale fibers with a noise-sensitive nonlinear process.

"Waves that appear from nowhere and disappear without a trace" is a fundamental nature for RWs [3]. The optical RWs have the same nature [4,5], which is often noted as "modulation instability" [6].

In recent years, the RWs in optical fiber system have attracted more attentions [7–9]. In consideration of that the RWs have a wide application prospect in the fields of optical communication, photonic computing and optical information processing, the researches related the optical RWs have been developing rapidly [10,11].

Breather homoclinic limit method is often applied to obtain the first-order RW solutions for nonlinear evolution equations (NLEEs). The RW solutions are derived from the limit of Akhmediev-breather (periodic in space but localized in time), Kuznetsov–Ma-breather (periodic in time but localized in space), and Peregrine-breather or generalized breather (time and space homoclinic) [12–14]. The Hirota bilinear method is often used to construct breather and other soliton solutions [15–21].

The well-known Darboux transformation is an effective method to construct RW solutions for NLEEs [22–27]. Its main idea is that NLEE is first transformed into its Lax representation, then by a series of transformations, RW solutions can be

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https://doi.org/10.1016/j.ijleo.2017.12.114
0030-4026/

computed algebraically with the obvious seed solution. The Darboux transformation is suitable to construct the first-order, second-order, even higher order RW solutions [28–33].

The nonlinear Schrödinger equation (NLSE) is one of the most fundamental mathematical models to describe the optical wave/pulse propagation in nonlinear optical fiber system [34–43].

In real optical fiber system, there always exist some non-uniformities owing to various nonlinear factors, such as imperfections of manufacture, variations in lattice parameters of fiber media, fluctuations in fiber diameters and so on. These non-uniformities often produce fiber gain/loss, phase modulation and variable dispersion [44–46]. The variable coefficient NLSEs (vcNLSEs) are effective models to describe the inhomogeneous effects of nonlinear optical pulse propagations. The vcNLSEs permit to reveal more abundant optical wave characteristics under various conditions and complex environments, such as varying group velocity dispersion (GVD), Kerr nonlinearity and system gain/loss. Unlike constant coefficient NLSEs (ccNLSEs), the studies on vcNLSEs show that one can excite and control the localized structures through their inhomogeneity parameters. Even though identifying and controlling the RWs in vcNLSE systems have been investigated by several authors [47–53]. There are still a great amount of unknown and valuable problems to be explored for the RWs of vcNLSEs.

Due to the computation complexity, it is difficult to obtain the RW solutions for the vcNLSEs. Similarity transformation is suitable to establish a bridge between the vcNLSEs and the ccNLSEs [47,53–55]. Owing to that the RW solutions of the ccNLSEs have been obtained early by pioneers, the RW solutions of the vcNLSEs may be constructed under some constraint conditions.

In this paper, we consider the following generalized vcNLSE in fiber system [46]

$$i\frac{\partial u}{\partial x} + i\beta_1(x)\frac{\partial u}{\partial t} + \beta_2(x)\frac{\partial^2 u}{\partial t^2} + \gamma(x)\left|u^2\right|u = 0, \tag{1}$$

where $u(x, t)$ represents space-time complex envelope of the electric field in fiber system, x is the longitudinal coordinate, and t is time in moving coordinate system. $\beta_1(x)$ and $\beta_2(x)$ represent different GVD coefficients, and $\gamma(x)$ is nonlinearity coefficient. Eq. (1) can degenerate to the standard vcNLSE while $\beta_1(x)$=0, and some soliton characteristics on the standard vcNLS have been discussed, such as oscillating solitons by the bilinear method [56].

We implement the similarity transformation to reduce Eq. (1) to a standard ccNLSE, and obtain the first-order and second-order RW solutions for Eq. (1), while a compatible condition is established, in which the both of $\beta_1(x)$, $\beta_2(x)$, $\gamma(x)$ are free functions. Choosing $\beta_1(x)$ and $\beta_2(x)$ as specific function, we excite several RWs, such as single-peak, two-peak and multiple-peak RW, double U-type RW. By setting parameters, we examine the controllability of the RWs for Eq. (1).

2. The analytic RW solutions for Eq. (1)

For Eq. (1), we introduce similarity transformation $u(x, t) = \lambda(x)U(X, T)\exp(i\varphi(x, t))$, where $T = T(x, t)$ is similarity transformation function, $X = X(x)$ is displacement function, $\lambda(x)$ is amplitude function, $\varphi(x, t)$ is phase function [54,55,47]. We assume that this transformation can reduce Eq. (1) to the following ccNLSE

$$iU_X - U_{TT} - 2\left|U\right|^2 U = 0. \tag{2}$$

Through computation, we have

$$r_0(x) = \left[c_0 + 2\int_0^x \beta_2(z)dz\right]^{-1}, \tag{3}$$

$$r_1(x) = c_1 r_0(x)\left[c_0 - \int_0^x \beta_1(z)dz\right], \tag{4}$$

$$r_2(x) = -\int_0^x \left[\beta_1(z)r_1(z) + \beta_2(z)r_1^2(z)\right]dz + c_2, \tag{5}$$

$$T_1(x) = s_0 c_0 r_0(x), \tag{6}$$

$$T_2(x) = -s_0 c_0 \int_0^x \left[\beta_1(z) + 2\beta_2(z)r_1(z)\right] r_0(z)dz + T_0, \tag{7}$$

$$\lambda(x) = \lambda_0[c_0 r_0(x)]^{1/2}, \tag{8}$$

$$\varphi(x, t) = \frac{t^2}{2}r_0(x) + tr_1(x) + r_2(x), \tag{9}$$

$$T(x, t) = tT_1(x) + T_2(x), \tag{10}$$

$$X(x) = -s_0^2 c_0 r_0(x)\int_0^x \beta_2(z)dz + X_0, \tag{11}$$