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Combined optical solitary waves and conservation laws for nonlinear Chen–Lee–Liu equation in optical fibers

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ABSTRACT

This paper obtains a combined optical solitary wave solution that is modeled by nonlinear Chen–Lee–Liu equation (NCLE) which arises in the context of temporal pulses along optical fibers associated with the self-steepening nonlinearity using the complex envelope function ansatz. The novel combined solitary wave describes bright and dark solitary wave properties in the same expression. The intensity and the nonlinear phase shift of the combined solitary wave solution are reported. Moreover, the Lie point symmetry generators or vector fields of a system of partial differential equations (PDEs) which is acquired by transforming the NCLE to a real and imaginary parts are derived. It is observed that the obtained system is nonlinearly self-adjoint with an explicit form of a differential substitution satisfying the nonlinear self-adjoint condition. Then we use these facts to establish a set of conservation laws (Cls) for the system using the general Cls theorem. Numerical simulation and physical interpretations of the obtained results are demonstrated with interesting figures showing the meaning of the acquired results. It is hoped that the results reported in this paper can enrich the nonlinear dynamical behaviors of the NCLE.

1. Introduction

It is well known that the investigation of solitary waves for the Nonlinear models has made a great progress [1–49]. The construction of optical solitary waves are not at a test phase rather than a reality. The solutions of these equations plays an important role in the study of nonlinear phenomena. The term “combined solitary wave” refers to a solitary wave that describe bright and dark solitary waves properties in the same expression and their amplitude may approach nonzero when the time variable approaches infinity [1]. In optical fiber contexts, the generation of the combined solitary waves in the normal dispersion regime was first reported by Li et al. [1]. Since its introduction into the literature, the solitary wave solutions of several models have been investigated [2–6]. Cls are important in the solving and reducing PDEs. The Cls assume that certain physical nature such as measurable quantities do not change in the course of time with an isolated system [51].

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Many powerful methods have been developed for the construction of Cls, some of which include the multiplier approach introduced by Bluman [51] and the general Cls theorem introduced by Ibragimov [52,53]. The NCLE is given by [7–11]:

$$i\psi_t + a\psi_{xx} + ib|\psi|^2\psi_x = 0. \tag{1}$$

The model has a lot of applications in plasma physics and nonlinear optical fibers [7]. Eq. (1) collapses to the regular NCLE when $a = b = 1$. In Eq. (1), the term b is associated with self-steepening phenomena and a represents the group velocity dispersion (GVD). The independent variables t and x represent the retarded time and propagation distance [8]. The NCLE equation is an integrable system that has been shown in [9]

In [7], the rogue wave solutions of the NCLE with higher order terms were reported. The traveling wave approach was utilized to retrieve the chirped singular optical soliton solutions in [8]. Further more, Triki et al. [10] retrieved the dark and gray type soliton solutions by using an ansatz technique. Finally in [11], the chirped W-shaped solitary waves of the model were reported.

In this work we will investigate the existence of combined optical solitary wave solution in nonlinear media which is governed by the NCLE for the first time. We will use the complex envelope function ansatz [1] to derive the closed form of dark-bright solitary wave solution. The paper conclude by reporting the Lie symmetries and Cls of the NCLE.

1.1. Combined optical solitary waves

We begin by assuming a solution of the form [1]:

$$\psi(x, t) = A(x, t) \times e^{i\phi(x,t)}, \tag{2}$$

where

$$\phi(x, t) = -kx + \omega t + \theta. \tag{3}$$

In Eq. (2), $A(x, t)$ is the complex envelope function and ϕ is the linear phase shift function. In Eq. (3), the parameters k is the phase components representing the wave numbers and ω represents the parameter of frequency shift, respectively, while θ is the phase constant.

The complex envelope ansatz [1] requires a modification of the solution in the form

$$A(x, t) = i\beta + \lambda\ \tanh[\eta(x - vt)] + i\rho\ \mathrm{sech}[\eta(x - vt)], \tag{4}$$

where η and v are the pulse width and velocity, respectively. In the case where $\beta = \lambda = 0$, we obtain bright solitary wave. But when $\rho = 0$, the solution given in Eq. (4) transforms to dark solitary wave solution. The presence of the parameters β, λ, ρ permits the ansatz Eq. (4) to describe a combined solitary wave solution [1]. The parameters η, v, k, ω are real values, but the parameter β, λ, ρ can be real or complex numbers depending on the equation parameters a and b [4]. Accordingly, the amplitude function of $A(x, t)$ can be written as

$$|A(x, t)| = \{\lambda^2 + \beta^2 + 2\ \beta\rho\ \mathrm{sech}[\eta(x - vt)] + (\rho^2 - \lambda^2)\mathrm{sech}^2[\eta(x - vt)]\}^{\frac{1}{2}}, \tag{5}$$

and its corresponding nonlinear phase shift function ψ_{NL} is in the form

$$\psi_{NL} = \arctan\left[\frac{\beta + \rho\ \mathrm{sech}[\eta(x - vt)]}{\lambda\ \tanh[\eta(x - vt)]}\right]. \tag{6}$$

Substituting Eq. (2) into Eq. (1) and removing the exponential term, we obtain

$$A(ak^2 + \omega - bk|A|^2) + i(2ak - b|A|^2)A_x - aA_{xx} - iA_t = 0. \tag{7}$$

By substituting Eq. (4) into Eq. (7), we get

$$\begin{aligned}
&iak^2\beta - ibk\beta^3 - ibk\beta\lambda^2 + i\beta\omega + iak^2\rho\ \mathrm{sech}(\tau) - 3ibk\beta^2\rho\ \mathrm{sech}(\tau) - ia\eta^2\rho\ \mathrm{sech}(\tau)\\
&-ibk\lambda^2\rho\ \mathrm{sech}(\tau) + i\rho\omega\ \mathrm{sech}(\tau) + 2iak\eta\lambda\ \mathrm{sech}(\tau)^2 + iv\eta\lambda\ \mathrm{sech}(\tau)^2 - ib\beta^2\eta\lambda\ \mathrm{sech}(\tau)^2\\
&+ibk\beta\lambda^2\ \mathrm{sech}(\tau)^2 - ib\eta\lambda^3\ \mathrm{sech}(\tau)^2 - 3ibk\beta\rho^2\ \mathrm{sech}(\tau)^2 + 2ia\eta^2\rho\ \mathrm{sech}(\tau)^3 - 2ib\beta\eta\lambda\rho\\
&\mathrm{sech}(\tau)^3 + ibk\lambda^2\rho\ \mathrm{sech}(\tau)^3 - ibk\rho^3\ \mathrm{sech}(\tau)^3 + ib\eta\lambda^3\ \mathrm{sech}(\tau)^4 - ib\eta\lambda\rho^2\ \mathrm{sech}(\tau)^4 + ak^2\lambda\\
&\tanh(\tau) - bk\beta^2\lambda\tanh(\tau) - bk\lambda^3\tanh(\tau) + \lambda\omega\tanh(\tau) + 2ak\eta\rho\ \mathrm{sech}(\tau)\tanh(\tau) + v\eta\rho\\
&\mathrm{sech}(\tau)\tanh(\tau) - b\beta^2\eta\rho\ \mathrm{sech}(\tau)\tanh(\tau) - 2bk\beta\lambda\rho\ \mathrm{sech}(\tau)\tanh(\tau) - b\eta\lambda^2\rho\ \mathrm{sech}(\tau)\tanh(\tau)\\
&+2a\eta^2\lambda\ \mathrm{sech}(\tau)^2\tanh(\tau) + bk\lambda^3\ \mathrm{sech}(\tau)^2\tanh(\tau) - 2b\beta\eta\rho^2\ \mathrm{sech}(\tau)^2\tanh(\tau) - bk\lambda\rho^2\ \mathrm{sech}(\tau)^2\\
&\tanh(\tau) + b\eta\lambda^2\rho\ \mathrm{sech}(\tau)^3\tanh(\tau) - b\eta\rho^3\ \mathrm{sech}(\tau)^3\tanh(\tau) = 0,
\end{aligned} \tag{8}$$