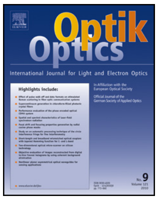
Optik 158 (2018) 312–315

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo



Original research article

Chirped dark solitons in optical metamaterials

Houria Triki[a], Anjan Biswas[b,c,d], M.M. Babatin[c], Qin Zhou[e,*]

[a] Radiation Physics Laboratory, Department of Physics, Faculty of Sciences, Badji Mokhtar University, P.O. Box 12, 23000 Annaba, Algeria
[b] Department of Physics, Chemistry and Mathematics, Alabama A&M University, Normal, AL 35762, USA
[c] Department of Mathematics and Statistics, College of Science, Al-Imam Mohammad Ibn Saud Islamic University, Riyadh 13318, Saudi Arabia
[d] Department of Mathematics and Statistics, Tshwane University of Technology, Pretoria 0008, South Africa
[e] School of Electronics and Information Engineering, Wuhan Donghu University, Wuhan 430212, PR China

ARTICLE INFO

Article history:
Received 27 November 2017
Accepted 24 December 2017

Keywords:
Solitons
Metamaterials

ABSTRACT

The ultrashort pulse propagation in metamaterials that is governed by the generalized nonlinear Schrödinger equation with pseudoquintic nonlinearity and self-steepening effect is investigated. By adopting a nonlinear chirp anstaz, we obtain an elliptic differential equation with a fifth-degree nonlinear term describing the evolution of the wave amplitude in the metamaterial. We present a variety of exact chirped dark soliton solutions for the model in the presence of the self-steepening term. The obtained results show that the corresponding chirp is proportional to the field intensity and depends on the self-steepening parameter. Parametric conditions on the metamaterial parameters for the existence of the dark structures as well as the nonlinear chirp associated with each of these optical pulses are also presented.

1. Introduction

The study of bright, dark and singular optical solitons has been going on for the past few decades [1–13]. This research is mostly focused on the propagation of such solitons through optical fibers and crystals. Later, the dynamics of soliton propagation through another form of waveguide gained popularity. This is the PCF. Subsequently, this yielded to the study of solitons through optical metamaterials (MMs). These form of waveguiudes carry great advantage over optical fibers and PCF. One minor factor is that MMs eliminate the issue of pulse splitting that leads to DGD and birefringence.

Motivated by the research taking place with solitons in optical MMs, we present for the first time to our knowledge chirped dark solitons for a generalized NLSE with higher-order effects such as pseudoquintic nonlinearity and self-steepening effect. We further find the corresponding chirping and show that the latter is directly proportional to the field intensity. The parametric conditions on the metamaterial parameters, under which dark structures can form, are also obtained.

This paper is organized as follows: In Section 2, we derive the dynamical equation governing the evolution of field amplitude by using a nonlinear chirp ansatz. In Section 3, we present a variety of exact analytical dark soliton solutions with nonlinear chirp of the model for different parameter conditions. Concluding remarks are given in Section 4.

---

* Corresponding author.
E-mail address: qinzhou@whu.edu.cn (Q. Zhou).

https://doi.org/10.1016/j.ijleo.2017.12.130

2. Theoretical model

The generalized nonlinear Schrödinger equation describing the propagation of ultrashort pulses in MMs governed by a pseudoquintic nonlinearity and self-steepening effect is written in the following form [1]

$$i\frac{\partial\psi}{\partial z}+\frac{k_2}{2}\frac{\partial^2\psi}{\partial t^2}+p_3|\psi|^2\psi-p_5|\psi|^4\psi-is_1\frac{\partial}{\partial t}(|\psi|^2\psi)=0, \tag{1}$$

where $\psi(z, t)$ is the complex envelope of the electric field, $t=cT/\lambda_p$ and $z=Z/\lambda_p$ are the respective normalized time and propagation distance, where λ_p is the plasma wavelength. k_2 stands for the group-velocity dispersion (GVD), while p_3, p_5 and s_1 denote the cubic, pseudoquintic nonlinearity and the self-steepening effect, respectively.

Exact bright, dark, and bright-gray quasi-soliton solutions of this equation have been obtained in the absence of the self-steepening term ($s_1=0$) [1]. It is of interest to investigate the propagation properties of envelope solitons in the presence of self-steepening effect since the latter will essentially influence the physical features of propagating pulses. We mention that the self-steepening, otherwise called the Kerr dispersion, is due to the intensity dependence of group velocity [2]. Remarkably, the effect of self-steepening on optical solitons is an important issue in the optical fiber communication system when the soliton pulse width becomes ultrashort <100 fs and subsequently the higher-order effects cannot be neglected [3,4].

To start with, we seek for the complex envelope traveling-wave solutions of Eq. (1) by taking [5–10]

$$\psi(z,t)=\rho(\xi)e^{i[\chi(\xi)-kz]}, \tag{2}$$

where $\xi=t-uz$ is the traveling coordinate, while ρ and χ are real functions of the traveling coordinate ξ. Here $u=1/v$, with v the group velocity of the wave packet. The corresponding chirp is given by $\delta\omega(t,z)=-\frac{\partial}{\partial t}[\chi(\xi)-kz]=-\chi'(\xi)$. By inserting Eq. (2) in Eq. (1) and separating out the real and imaginary parts of the equation, we find the pair of coupled equations in ρ and χ,

$$k\rho+u\chi'\rho-\frac{k_2}{2}\chi'^2\rho+\frac{k_2}{2}\rho''+s_1\chi'\rho^3+p_3\rho^3-p_5\rho^5=0, \tag{3}$$

and

$$-u\rho'+\frac{k_2}{2}\rho\chi''+k_2\rho'\chi'-3s_1\rho^2\rho'=0. \tag{4}$$

Now we adopt an *ansatz* that depends quadratically on the wave amplitude as

$$\chi'=p\rho^2+q, \tag{5}$$

where p and q are the nonlinear and constant chirp parameters, respectively. Accordingly, the resultant chirp takes the form: $\delta\omega(t,z)=-(p\rho^2+q)$. This implies that the chirp associated with propagating pulses is intensity dependent and includes both linear and nonlinear contributions. Further substitution of the *ansatz* (5) into Eq. (4) gives the relations of q and p as

$$p=\frac{3s_1}{2k_2},\quad q=\frac{u}{k_2}. \tag{6}$$

As seen from the first relation of (6), the nonlinear chirp parameter depends on the self-steepening coefficient s_1. Therefore, we can conclude that the origin of the nonlinear chirp comes from the higher-order nonlinear effects such as the self-steepening effect.

Now, using Eqs. (5) and (6) in Eq. (3), one obtains

$$\rho''+\delta\rho+\beta\rho^3+\gamma\rho^5=0, \tag{7}$$

where

$$\delta=\frac{2k_2k+u^2}{k_2^2},\quad \beta=\frac{2(k_2p_3+us_1)}{k_2^2},\quad \gamma=\frac{3s_1^2-8k_2p_5}{4k_2^2}. \tag{8}$$

Eq. (7) is an elliptic differential equation describing the evolution of the wave amplitude in the metamaterial. Generally speaking, Eq. (7) can be mapped into φ^6 field equation, which is well known to admit bright soliton, dark soliton, kink, and double kink solutions [5,11,12]. However, seeking more new exact soliton solutions of this equation is still an essential task in mathematical physics. In what follows, we report novel dark-type soliton solutions of this equation under different parametric conditions.

3. Chirped dark solitons

Here we present exact chirped dark soliton solutions of the model (1) in the presence of self-steepening term based on solving Eq. (7). The dark soliton structures and their corresponding chirping are discussed in the following four cases: