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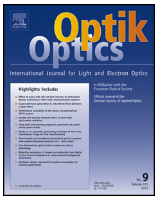


Original research article

# Optical solitons for complex Ginzburg–Landau model in nonlinear optics

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ABSTRACT

This paper studies the complex Ginzburg–Landau equation (CGLE) which models soliton propagation in the presence of detuning factor in nonlinear optics. Dark, bright, dark-singular and a new dark-bright optical soliton solutions to the model are derived using the sine-Gordon equation method (SGEM). Singular soliton solutions are also celebrated. The model is studied with Kerr law, quadratic–cubic law and parabolic laws nonlinear fibers. It is hoped that the results reported in this paper can enrich the nonlinear dynamical behaviors of the CGLE.



## 1. Introduction

Optical solitons have promising potential to become principal information carriers in telecommunication due to their capability of propagating long distance without attenuation and changing their shapes [1–3]. The CGLE that will be studied in this paper models soliton propagation in the presence of detuning factor [9]. There are a lot of models that describe the dynamics of soliton propagation through optical fibers, and many results are reported in the past few decades [5–52]. The integrability aspect will be the focus of this paper.

In this study, the CGLE will be examined. Three types of nonlinear fibers including quadratic–cubic law and Kerr law Parabolic laws nonlinearities will be studied. In order to integrate the equation for each type of nonlinearity, the SGEM [4–8] will be employed to achieve this task. This naturally lead to some constraints placed on the soliton parameters and are discussed in their respective sections.

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## 2. Theoretical model

The dimensionless form of the CGLE that will be studied in this paper is given by [9–13]:

$$i\psi_t + a\psi_{xx} + bF(|\psi|^2)\psi = \frac{1}{|\psi|^2\psi^*}\{\alpha|\psi|^2(|\psi|^2)_{xx} - \beta((|\psi|^2)_x)^2\} + \gamma\psi, \quad i = \sqrt{-1}, \tag{1}$$

where $x$ is the non-dimensional distance along the fiber and $t$ is the time in dimensionless form. $\psi(t, x)$ is the dependent variable. The constants $\alpha$, $\beta$ and $\gamma$ arise from the perturbation effect of the model, while $a$ and $b$ are from the group velocity dispersion (GVD). The function $F(|\psi|^2)\psi$ is a real-valued algebraic function and is $k$-times continuously differentiable [14], so that

$$F(|\psi|^2)\psi \in \sum_{m,n=1}^{\infty} C^k((-n, n)X(-m, m); \mathbb{R}^2). \tag{2}$$

Eq. (1) fails the Painleve test of integrability and thus it is not integrable using the classical methods such as the inverse scattering transform [10]. Eq. (1) was studied with Kerr law and power law nonlinearity in [9] using the functional variable, sine–cosine, Riccati-expansion, $(G'/G)$-expansion, extended trial equation, first integral, generalized Kudryashov, extended Jacobi elliptic function and F-expansion methods, optical and singular soliton solutions of the model were reported. Biswas [11] studied the model with power law nonlinearity using the He's semi-inverse variational principle and reported a 1-soliton solution to the model. In [12], the periodic solitary wave solutions of the model were constructed by the extended Jacobi elliptic function expansion method. The simplest equation method was applied used to study the model with Kerr law and power law nonlinear fibers [13], dark and singular soliton solutions were reported. Also, the variational iteration method was used to obtain the soliton solutions of the model [10]. By setting

$$\alpha = 2\beta. \tag{3}$$

Eq. (1) simplify to

$$i\psi_t + a\psi_{xx} + bF(|\psi|^2)\psi = \frac{\beta}{|\psi|^2\psi^*}\{2|\psi|^2(|\psi|^2)_{xx} - ((|\psi|^2)_x)^2\} + \gamma\psi. \tag{4}$$

To study Eq. (1), we apply the following transformation

$$\psi(t, x) = u(\xi) \times e^{i\phi(t,x)}, \quad \xi = (x - vt), \tag{5}$$

where

$$\phi = -kx + \omega t + \theta, \tag{6}$$

where $\phi(t, x)$ represents the phase component, $k$ is the wave number, $\omega$ represents the frequency, $\theta$ represents the phase constant, $v$ is the velocity of the soliton. Putting Eq. (5) into Eq. (4) and separating the into real and imaginary components, two equations are obtained. The imaginary component gives

$$v = -2ka, \tag{7}$$

and the real part gives

$$(-ak^2 - \gamma - \omega)U + bF(U^2)U + (a - 4\beta)U'' = 0. \tag{8}$$

In the subsequent sections, Eq. (8) will be analyzed in detail for the three forms of nonlinear fibers, i.e. Kerr law, quadratic–cubic law and parabolic law nonlinearity.

## 3. Description of the sine-Gordon expansion method

Consider the following sine-Gordon equation

$$\psi_{xt} = \alpha \sin(\psi), \tag{9}$$

where $\alpha$ is a non-zero constant. We apply the transformation

$$\psi(t, x) = U(\xi), \quad \xi = \eta(x + vt), \tag{10}$$

where $v$ is the traveling wave velocity. Substituting Eq. (10) into Eq. (9), we get

$$U'' = \frac{\alpha}{v\eta^2}\sin(U(\xi)). \tag{11}$$