Optik 158 (2018) 399–415

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Original research article

Optical soliton perturbation for complex Ginzburg–Landau equation with modified simple equation method

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ARTICLE INFO

Article history:
Received 25 November 2017
Accepted 24 December 2017

OCIS Codes:
060.2310
060.4510
060.5530
190.3270
190.4370

Keywords:
Solitons
Perturbation
Full nonlinearity
Modified simple equation equation method

ABSTRACT

The modified simple equation method applied to perturbed complex Ginzburg–Landau equation gives dark and singular solutions to the model. The perturbation terms appear with full nonlinearity. There are eight nonlinear forms studied in this paper. These solitons appear with constraint conditions that gurantee its existence and these are also presented.

1. Introduction

The complex Ginzburg–Landau equation (CGLE) is one of the many models that studies the dynamics of optical soliton propagation through a wide range of waveguides such as crystals, optical fibers, optical couplers, optical metamaterials and metasurfaces as well as PCF. This model is an extended version of the usual nonlinear Schrödinger's equation that is visible all across. These models and other nonlinear evolution equations, that arise in mathematical photonics as well as other areas of mathematical physics, are all successfully addressed by the modified simple equation method [1–10]. When perturbation terms are turned on, the perturbed CGLE will be addressed in this paper by the modified simple equation method. There are eight forms of nonlinear media that will be studied. All of these nonlinear forms will lead to the retrieval of dark and singular

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https://doi.org/10.1016/j.ijleo.2017.12.131
0030-4026/

soliton solutions to the governing model. The existence of these solitons will be guaranteed with the constraint conditions in the parameters. The perturbation terms are all of Hamiltonian type. After a quick recapitulation of this integration scheme, soliton solutions will be derived in the subsequent subsections.

1.1. The model

The dimensions form of CGLE is [2–4,6–8]:

$$iq_t + aq_{xx} + bF\left(|q|^2\right)q = \frac{1}{|q|^2 q^*}\left[\alpha|q|^2\left(|q|^2\right)_{xx} - \beta\left\{\left(|q|^2\right)_x\right\}^2\right] + \gamma q \tag{1}$$

where x is the spatial variable that represents the non-dimensional distance along the fiber, while t is the temporal variable. Then, a, b, α, β and γ are valued constants. The coefficients a and b come from the group velocity dispersion and nonlinearity, respectively. The terms with α and β are additional nonlinear terms and γ comes from detuning effect.

In (1), F is real-valued algebraic function and it is necessary to possess the smoothness of the complex function $F\left(|q|^2\right)q$ is k times continuously differentiable, so that

$$F\left(|q|^2\right)q \in \bigcup_{m,n=1}^{\infty} C^k((-n,n) \quad \times \quad (-m,m); \quad R^2).$$

In presence of perturbation terms, CGLE gets extended to [4]:

$$iq_t + aq_{xx} + bF\left(|q|^2\right)q = \frac{1}{|q|^2 q^*}\left[\alpha|q|^2\left(|q|^2\right)_{xx} - \beta\left\{\left(|q|^2\right)_x\right\}^2\right] + \gamma q + i\left[\delta q_x + \lambda\left(|q|^{2m}q\right)_x + \mu\left(|q|^{2m}\right)_x q\right] \tag{2}$$

where δ is the inter-modal dispersion that arises in addition to chromatic dispersion, λ represents the self-steepening effect for short pulses and μ is the higher–order dispersion coefficient. The parameter m accounts for full nonlinearity.

2. Revisitation of the integration algorithm

Suppose we have a nonlinear evolution equation in the form [1,5,9,10]:

$$P(u, u_t, u_x, u_{tt}, u_{xt}, u_{xx}, \ldots) = 0 \tag{3}$$

where P is a polynomial in $u(x, t)$ and its partial derivatives in which the highest order derivatives and nonlinear terms are involved. In the following, we give the main steps of this method.

Step-1: We use the transformation

$$u(x,t) = U(\xi), \quad \xi = x - ct, \tag{4}$$

where c is a constant to be determined, to reduce Eq. (3) to the following ODE:

$$Q\left(U, U', U'', U''', \ldots\right) = 0 \tag{5}$$

where Q is a polynomial in $U(\xi)$ and its total derivatives, while $' = d/d\xi$.

Step-2: We suppose that Eq. (5) has the formal solution

$$U(\xi) = \sum_{l=0}^{N} a_l \left(\frac{\psi'(\xi)}{\psi(\xi)}\right)^l, \tag{6}$$

where a_l are constants to be determined, such that $a_N \neq 0$, and $\psi\left(\xi\right)$ is an unknown function to be determined later.

Step-3: We determine the positive integer N in Eq. (6) by considering the homogeneous balance between the highest order derivatives and the nonlinear terms in Eq. (5).

Step-4: We substitute (6) into (5), then we calculate all the necessary derivatives $U', U'', \ldots$ of the unknown function $U(\xi)$ and we account the function $U(\xi)$. As a result of this substitution, we get a polynomial of $\psi'\left(\xi\right)/\psi\left(\xi\right)$ and its derivatives. In this polynomial, we gather all the terms of the same power of $\psi^{-j}\left(\xi\right)$, $j = 0, 1, 2, \ldots$ and its derivatives, and we equate with zero all the coefficients of this polynomial. This operation yields a system of equations which can be solved to find a_k and $\psi\left(\xi\right)$. Consequently, once can retrieve the exact solutions of Eq. (3).

3. Soliton solutions

In order to solve Eq. (2) by the trial equation method, we start with the following wave transformation

$$q(x,t) = U(\xi)e^{i\phi(x,t)}, \quad q^*(x,t) = U(\xi)e^{-i\phi(x,t)} \tag{7}$$

where $U(\xi)$ represents the shape of the pulse, $\xi = x - vt$ and $\phi = -\kappa x + \omega t + \theta$. The function $\phi(x, t)$ is the phase component of the soliton, κ is the soliton frequency, while ω is the wave number, θ is the phase constant and v is the velocity of the soliton.