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# An accurate approach for the computation of polar harmonic transforms

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ABSTRACT

Polar Harmonic Transforms (PHTs) can be used as a feature extraction technique by exploiting their orthogonal basis that generate a set of rotation invariant features. PHTs have distinct traits as compared with other transforms and moments. This is because of their less redundant information. Applied to digital images polar harmonic transforms suffer from various numerical and statistical errors. In this paper a computational framework based on two different numerical integration schemes which reduce the numerical errors in the computation of the transforms. Extensive experiments are carried out in order to assess the accuracy of the proposed technique in face recognition problem. Two standard face databases are employed, i.e., ORL and JAFFE, which contain images with different variations. Furthermore, the rotation invariant trait of the proposed approach is evaluated using different rotation angles. The proposed approach attenuates the numerical errors as well as provides numerical stability to PHTs of high orders.



## 1. Introduction

Moments and transforms are known to be excellent image descriptors. Through image moments and transforms we can easily found many simple properties of image which include area (or total intensity), its centroid, and information about its orientation symmetry structure. Moments and transforms have a wide range of applications in image analysis, such as, invariant pattern or object recognition, image reconstruction, content-based image retrieval, watermarking, data-hiding, optical character recognition, pattern classification, face recognition, map matching, palm print verification and edge detection. The capability of image moments and transforms of feature representation have been widely used in object identification techniques in several areas of computer vision and robotics. Information about the different types of geometrical features of the image is provided by the set of moments and transforms computed from digital image. Moreover, the moment and transform descriptor are simple to use and can be designed to provide fast, efficient and versatile system to calculate numerical features for many applications. The accuracy of pattern recognition process can be enhanced by using the appropriate feature extraction method. One of the oldest moment and transform based shape descriptors, which are used to generate a set of invariants that have been used in many pattern recognition processes, are Geometric moments (GMs). The major disadvantage of GMs is that their information redundancy is very high. On the other hand, orthogonal radial moments and transforms are free from these drawbacks because each moment and transform describes distinct information of an image.

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Among the orthogonal radial moments, Zernike moments (ZMs), Pseudo Zernike moments (PZMs) and orthogonal Fourier Mellin moments (OFMMs) are generally used to represent an image with minimum amount of information redundancy. Orthogonal transforms include polar harmonic transforms (PHTs). PHTs consist of polar complex exponential transforms (PCETs), polar cosine transforms (PCTs) and polar sine transforms (PSTs). The main advantage of PHTs is low computational complexity over ZMs, PZMs. and OFMMs. Transformations are numerically instable at low frequency transformation and numerically stable at high frequency of transforms. The radial function of moments (ZMs, PZMs, OFMMs) are polynomial in $r$, whereas the radial function of transforms are harmonic in nature, i.e., they are basic waves.

Moment invariant was first introduced by Hu [1] in 1962. He employed the results of the theory of algebraic invariants and derived his seven famous invariants to rotation of 2-D objects. At that time, hundreds of papers have been devoted to various improvements, extensions and generalizations of moment invariants and also to their use in many areas of application. Moment invariants have become one of the most important and most frequently used shape descriptors. Even though they suffer from certain intrinsic limitations, which prevent direct utilization for occluded object recognition, they frequently serve as first choice descriptors and as a reference method for evaluating the performance of other shape descriptors. Despite a tremendous effort and huge number of published papers, many open problems remain to be resolved. Teague [2] present orthogonal Zernike moments to image analysis, ZMs possess many useful characteristics as they provide good quality of image representation due to less sensitive to noise. Due to the orthogonal property ZMs possess minimum information redundancy as they belong to a set of orthogonal and complete polynomials defined inside a unit circle. They are found to be robust to image noise. These features of ZMs, are very useful in image description, but they also suffer from various errors which affect their accuracy. Bhatia and wolf [3] introduce class of circular orthogonal moments called Pseudo Zernike moments (PZMs), which exhibit similar properties to ZMs. Another important orthogonal rotation invariant moment known as OFMMs was first introduced by Sheng and Shen [4]. Apart from the characteristics of moments they suffer from various errors which affect their accuracy as well immunity to noise. The two major errors which arise in moment computation are geometric error and numerical integration error. These errors are also found in all ORIMs. The researchers observed that geometric error basically arises when a rectangular or square image is mapped geometrically inside the unit circle. This mapping allows only those pixels into computation of moments whose center fall inside the unit circle while discard the rest. As a result, the total pixel area used for the moment computation becomes unequal to the area of the circular region. The second error, that is, numerical integration error is caused when double integration in moment computation is approximated by zeroth order summation. Many algorithms have been developed till now for the removal of these errors. In term of noise sensitivity OFMMS have better performance than ZMs and PZMs for small images. Ping and Sheng [5] introduced Chebyshev-Fourier moments (CHFMS). Polar harmonic transforms (PHTs) are recently proposed by Yap et al. [6], which are similar to ORIMs. They observe that PHTs can be used to generate rotation-invariant features. The computation of PHTs kernel function is simple as compare to ZMs and PZMs. PHTs are numerical stable so numerical stability issue is nonexistent. The authors compared the kernel generation complexibility of PHTs with popular ZMs and PZMs. They observed that kernel generation is much less complex as compare to ZMs and PZMs.

PHTs are one of such image descriptors, which are robust to image noise, invariant with respect to rotation, and after certain geometric transformations they can be made scaling and translation invariant [7]. The computation complexity of PHTs is also very low as compared to ORIMs [8]. Due to the orthogonal property, PHTs exhibit many useful characteristics such as they possess minimum information redundancy and immunity to noise. PHTs are popular with various applications such as Fingerprint classification [9], Pattern recognition [10], Character recognition, Image reconstruction [11], Image retrieval, Image data mining, face recognition [12] Texture classification, Corneal surface modeling, Biomedical imaging such as MRI or PET cans(using the 3D formulation), etc. [13,14]. The accuracy of these applications depends on the accuracy of the moment values. Therefore, the accurate computation of transforms is the major task for these applications. PHTs suffer from various errors. These errors affect the accuracy of results. Many approaches have been proposed to remove these errors till now, which reduced the errors but not completely. We propose two novel methods for the accurate computation of PHTs. The first method is based on grid division method and second method developed using wavelet integration. In the first approach, when we take average value of all sub regions, the errors involved tend to reduce simultaneously. In this approach we avoid all those sub regions whose center falls outside the circle. The angular polynomials of transforms are computed recursively, which takes less time as compare to direct method. Another important advantage of using this method is that its calculations are simple as compare to previous methods. This method gives accurate results as the pixels are divided into sub regions which means every small region will be covered whose center falls inside the circle. For high order transforms, as the reconstruction error decreases in reconstructed images, the distortion present at the center of circle also decreases and hence accurate reconstructed image is obtained. The method also increases the speed of computation which is affected by computation of angular polynomials. The second method using wavelet integration is also effective technique for the computation of polar harmonic transforms. The PHTs have well established applications in face recognition problems. To strengthen our claim of accuracy some experiments are performed on different face databases. The experiments reveal that the proposed methods give high recognition rate as compared to the existing approaches.

The rest of the presented paper is ordered as follows: Section 2 provides an overview of concept of the PHTs, while their computational framework is discussed in Section 3. Section 4 presented detailed of the experiments. Conclusions are presented in Section 5.