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An alternative method for determining the energy of a photon

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ABSTRACT

In this paper a simple theoretical analysis that confirms the Einstein's hypothesis of the light quantum energy is presented. Contrary to the Einstein's thermodynamic approach to the black–body radiation and its analogy to the ideal gas, our analysis starts from the assumption that the

electromagnetic radiation is quantized. Based on the first principle and the Lorentz transformations properties we found that photon energy can be only directly proportional to the frequency of the electromagnetic radiation.

1. Introduction

Almost two decades passed before the Einstein's light quanta hypothesis [1] was accepted. The period of transition between the hypothesis rejection and acceptance would undoubtedly have been longer if Compton's experiment had been less striking [2]. Based on the thermodynamic approach to the black–body radiation, Einstein showed that monochromatic black–body radiation in the Wien's law spectral region behaves with respect to thermal phenomena as if it consists of independently moving particles or quanta of radiant energy. It was also shown that the energy of each quantum is proportional to its frequency.

In this paper we will try to find another approach in order to determine the photon energy. The only assumption that will be taken into consideration in the further analysis is that the electromagnetic radiation is quantized where the photon represents the radiation quantum. By starting from this assumption and by including the Lorentz transformation properties into the analysis we will find that in the frame of the first principle the only possible solution for the energy of the photon is the one that is directly proportional to the electromagnetic radiation frequency.

2. Theoretical analysis

In order to find the mathematical relation that will define the dependence of the photon energy on the other relevant physical parameters, the theoretical analysis will be started only from the assumption that the electromagnetic radiation is quantized and the photon is the quantum of such an electromagnetic radiation. Since any wave motion $u(\mathbf{r}, t)$ is fully defined by its amplitude $U(\mathbf{r})$, wave number $\mathbf{k}$ and angular frequency ω as $u(\mathbf{r}, t) = U(\mathbf{r})\cos(\mathbf{k}\cdot\mathbf{r} - \omega t + \varphi)$, where, without losing the generality of the analysis, the influence of the initial wave phase φ will be neglected, we will assume that the photon, as an integral part of the electromagnetic wave (radiation), has the energy ε that is dependent on these three abovementioned parameters, i.e. $\varepsilon = \varepsilon(U, \mathbf{k}, \omega)$. If the initial phase affects the photon energy, the photon energy will be dependent on the choice of the initial time, which is in the collision with the energy conservation law and thus absurd. For the wave number

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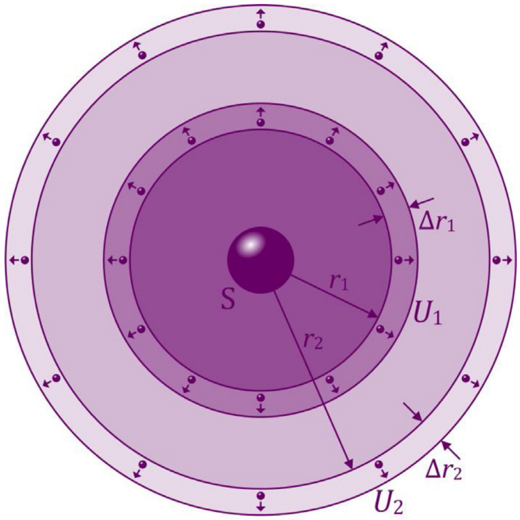

Fig. 1. Isotropic, unvarying, static, and monochromatic source S of the electromagnetic radiation.

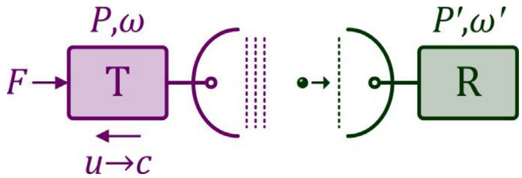

Fig. 2. Transmitter T emits planar, unvarying, and monochromatic electromagnetic waves in the direction of the receiver R.

we have $\left|\mathbf{k}\right| = 2\pi/\lambda$, where λ is the wavelength of the electromagnetic wave in vacuum and $\lambda = 2\pi c/\omega$ is valid, where c is the speed of light in vacuum, so the wave number is only dependent on the frequency $\mathbf{k} = \mathbf{k}(\omega)$. Therefore, for the photon energy we have $\varepsilon = \varepsilon(U, \omega)$.

In order to determine the dependence of the photon energy on the wave amplitude, we will observe the isotropic, unvarying, static, and monochromatic source S of the electromagnetic radiation, as it is depicted in Fig. 1. At the distance r_1 from the source the amplitude of the radiation is U_1 and at the distance r_2 the amplitude is U_2. As $r_1 \neq r_2$ we have also $U_1 \neq U_2$. Since the source is unvarying it emits constant power P of electromagnetic radiation, or equivalently it emits a constant number of photons per unit time q. In the analysis we will consider that the photon velocity is the same as the speed of light. If the photon velocity is smaller than the speed of light than some parts of an electromagnetic wave that was firstly broadcasted by the transmitter, will be not quantized at large distance from the transmitter and the others will be, which is absurd. Of course, it is also absurd that photons have the velocity greater than the speed of light because it is in the collision with the basic principle of the special relativity. So, after time $t_1 = r_1/c$ from the moment of their emission, photons reach the distance r_1 from the source The total number of photons, contained in a thin spherical shell with the thickness Δr_1 at the distance r_1, whereby $\Delta r_1 \ll r_1$ is satisfied, is $N_1 = q\Delta t_1$, where $\Delta t_1 = \Delta r_1/c$, with the assumption that there are no photons absorbed or scattered on their way since they are propagating in vacuum. Similarly, after time $t_2 = r_2/c$ from the moment of their emission, photons reach the distance r_2 from the source. Also, the total number of photons, contained in a thin spherical shell with the thickness Δr_2 at the distance r_2, whereby $\Delta r_2 \ll r_2$ is satisfied, is $N_2 = q\Delta t_2$ where $\Delta t_2 = \Delta r_2/c$, with the same assumption that there are no photons absorption and/or scattering on their way.

Based on the continuity equation for $\Delta t_1 = \Delta t_2 = \Delta t$ the following is valid $N_1 = N_2 = N$, or due to the invariability of the velocity of photons, all photons contained in the spherical shell defined by the distances $[r_1, r_1 + \Delta r]$ after time interval $\Delta t_{12} = (r_2 - r_1)/c$ are contained in the spherical shell defined by the distances $[r_2, r_2 + \Delta r]$, where $\Delta t = \Delta r/c$ is valid. The energy W_1 of all photons that are located in the spherical shell at a distance r_1 is $W_1 = N\varepsilon(U_1, \omega)$, while the energy W_1 of all photons that are located in the spherical shell at a distance r_2 is $W_2 = N\varepsilon(U_2, \omega)$. Based on the energy conservation law we have $W_1 = W_2$ that further leads to $\varepsilon(U_1, \omega) = \varepsilon(U_2, \omega)$, where it was considered that there are no external physical fields that can impact photons motion. As it is fulfilled $\varepsilon(U_1, \omega) = \varepsilon(U_2, \omega)$ for any, arbitrarily chosen, amplitudes U_1 and U_2, it can be concluded that the amplitude of the electromagnetic wave does not affect the photon energy, so we can finally conclude that the energy of the photons depends only on the frequency of electromagnetic waves, or $\varepsilon = \varepsilon(\omega)$.

In order to determine the mathematical relationship between the photon energy and the frequency of the electromagnetic wave we will consider the transmitter T that emits planar, unvarying, and monochromatic electromagnetic waves in the direction of the receiver R, as shown in Fig. 2. The radiated power and angular frequency of the transmitter electromagnetic