Optik 157 (2018) 180–183

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Original research article

Wave diffraction by a half-plane for grazing incidence

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ARTICLE INFO

Article history:
Received 19 September 2017
Accepted 13 November 2017

Keywords:
Diffraction theory
Grazing incidence
Half-plane

ABSTRACT

The diffracted wave, related with a half-plane on which the total field satisfies the Dirichlet boundary condition, becomes zero when the angle of incidence of the incident wave is equal to zero according to the solution of Sommerfeld. However, this behaviour of the solution causes some physical inconsistencies. The letter proposes a new approach for the elimination of the mentioned defect. The results are supported by some simulations.

1. Introduction

The diffraction of waves by a half-screen is a canonical problem the exact solution of which was first obtained by Sommerfeld [1]. Its' high-frequency asymptotic form constitutes the basis of the geometrical theory of diffraction (GTD) [2,3]. The diffraction field is a function of the angles of incidence and observation. Some physical problems require the consideration of the incidence angle a 0° [4,5]. This case is known as the grazing incidence and the GTD field becomes zero for the Dirichlet boundary condition (tangential component of the total electric field is zero on the surface of the scatterer). However, the incident wave will be discontinuous after it leaves the surface of the half-plane and its' discontinuity will not be compensated, because the diffracted field does not exist according to the literature [6]. When the boundary condition is Neumann (normal derivative of the total field is equal to zero on the surface of the scatterer), the value of the diffracted wave doubles. In this case the literature proposes the division of the diffracted field by two [7], but this suggestion cannot be supported physically or mathematically. This paper proposes the solution of the mentioned problem by using the diffraction field of GTD.

2. Theory

The geometry, in Fig. 1, is taken into account. The incident wave has the expression of $E_0\exp(jkx)\vec{e}_z$ where E_0 is the complex amplitude of the electric field. k is wave-number. x has the expression $\rho\cos\phi$ for the cylindrical coordinates are given by (ρ, ϕ, z). k is the wave-number. P shows the observation point.

The half-plane is perfectly electric conductor. Thus the tangential component of the total electric field is equal to zero on the surface. The edge of the half-screen is located on the z axis. The total electric field of the scattering problem by a perfectly conducting half-screen can be given by

$$\vec{E}_T = E_0 \left\{ e^{jk\rho\cos(\phi-\phi_0)} F\left[\xi_-\right] - e^{jk\rho\cos(\phi+\phi_0)} F\left[\xi_+\right] \right\} \vec{e}_z \tag{1}$$

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https://doi.org/10.1016/j.ijleo.2017.11.071

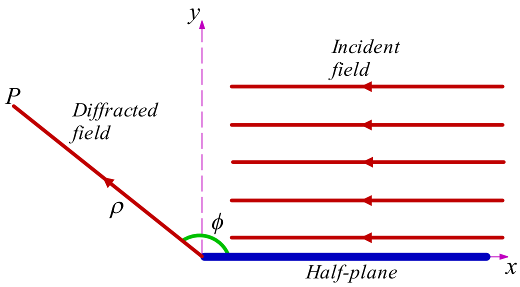

Fig. 1. Diffraction geometry by a half-plane.

according to the solution of Sommerfeld [1]. $F[x]$ is the Fresnel function, which can be defined by the integral

$$F\left[x\right]=\frac{e^{j\frac{\pi}{4}}}{\sqrt{\pi}}\int_{x}^{\infty}e^{-jv^2}dv. \tag{2}$$

$\xi_\pm$ is the detour parameter and has the expression $-(2k\rho)\cos[(\phi-\phi_0)/2]$. ϕ_0 shows the angle of incidence. It is apparent that the total field, in Eq. (1), vanishes when the angle of incidence is equal to zero. For the problem, defined by Fig. 1, the reflected field does not exist. However, there is not any mathematical or physical basis of considering only the first term, at the right-hand side of Eq. (1), in this case. Furthermore, the boundary condition cannot be satisfied if only this term is taken into account for the grazing incidence. When the incident wave passes the half-plane and enters the region $x<0$, it will have a discontinuity at $y=0$. This situation requires the existence of a diffracted field that will compensate the discontinuity of the geometrical optics (GO) wave and also propagate in the region $y<0$. This diffracted field must also satisfy the Dircihlet boundary condition of the surfaces of the half-screen. As can be seen from the above discussions, the actual form of the exact solution does not give a clue for the solution. Instead of Eq. (1), the high-frequency approximation, or GTD, form of the diffracted wave is taken into account. The field can be expressed by the equation

$$\vec{E}_d=-E_0\frac{e^{-j\frac{\pi}{4}}}{2\sqrt{2\pi}}\left(\frac{1}{\cos\frac{\phi-\phi_0}{2}}-\frac{1}{\cos\frac{\phi+\phi_0}{2}}\right)\frac{e^{-jk\rho}}{\sqrt{k\rho}}\vec{e}_z, \tag{3}$$

which also ceases to exist when the angle of incidence is zero [3]. The diffracted field can be written by

$$\vec{E}_d=\vec{E}_{d1}+\vec{E}_{d2} \tag{4}$$

where $\vec{E}_{d1}$ and $\vec{E}_{d2}$ can be introduced as

$$\vec{E}_{d1}=-\frac{E_0e^{-j\frac{\pi}{4}}}{4\sqrt{2\pi}}\left(\frac{\sin\frac{\pi+\phi-\phi_0}{4}}{\cos\frac{\pi+\phi-\phi_0}{4}}-\frac{\sin\frac{\pi-\phi-\phi_0}{4}}{\cos\frac{\pi-\phi-\phi_0}{4}}\right)\frac{e^{-jk\rho}}{\sqrt{k\rho}}\vec{e}_z \tag{5}$$

and

$$\vec{E}_{d2}=\frac{E_0e^{-j\frac{\pi}{4}}}{4\sqrt{2\pi}}\left(\frac{\sin\frac{\pi+\phi+\phi_0}{4}}{\cos\frac{\pi+\phi+\phi_0}{4}}-\frac{\sin\frac{\pi-\phi+\phi_0}{4}}{\cos\frac{\pi-\phi+\phi_0}{4}}\right)\frac{e^{-jk\rho}}{\sqrt{k\rho}}\vec{e}_z \tag{6}$$

respectively [7]. E_{d1} is related with the incident field, since it has an asymptote at $\pi+\phi_0$, which is the location of the shadow boundary. E_{d2} approaches to infinity at $\pi-\phi_0$. For this reason, it is related with the reflected wave. Note that E_{d1} and E_{d2} satisfy the Dirichlet boundary condition on the surfaces of the half-plane. For the geometry, in Fig. 1, only E_{d1} is considered, because the reflected field does not exist. We propose that the total electric field can be represented by

$$\vec{E}_T=E_0e^{jk\rho\cos\phi}U\left(\pi-\phi\right)\vec{e}_z+\vec{E}_d \tag{7}$$

where $U(x)$ is the unit step function, which is equal to one for $x>0$ and zero otherwise. E_d is equal to E_{d1} for ϕ_0 is equal to zero. The diffracted field satisfies the boundary condition on the surface of the half-screen. E_d has an asymptote at π, which is related with the first term, in Eq. (5). The asymptotes of the second term are located at $-\pi$ and 3π, which are out of the physical space of the problem. In order to eliminate the infinity, coming from the first term, the uniform theory of diffraction (UTD), will be used [6,8]. According to this theory, a transition function is introduced as

$$T\left(x\right)=\frac{F\left[|x|\right]}{\hat{F}\left[|x|\right]} \tag{8}$$