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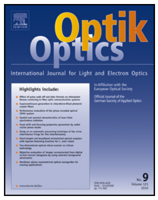

Original research article

Optical solitons for Biswas-Milovic Model in nonlinear optics by Sine-Gordon equation method

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A B S T R A C T

This article studies the dynamics of optical solitons to the Biswas–Milovic equation (BME) which describes the dynamics of optical solitons in generalized form. There are four types of nonlinear fibers studied in this paper. They are quadratic-cubic, Kerr and Parabolic laws. The integration algorithm adopted is the Sine-Gordon equation method (SGEM). Dark, bright and dark-singular optical solitons are derived. Complex singular solitons are also reported. The obtained results have important applications in the study of solitons in optical fibers.

1. Introduction

Nonlinear Schrödinger equations (NLSEs) appear in various areas of engineering sciences, physical and biological sciences. In particular, the NLSEs appears in fluid dynamics, nonlinear optics, plasma and nuclear Physics [1,2]. We observe many new progresses in the field of nonlinear optics [3–55]. The dimensionless form of the BME that will be studied in this paper is given by [3–13]:

$$i(|\psi^m)_t + a(|\psi^m)_{xx} + bF(|\psi|^2)|\psi^m = 0, \ \ i = \sqrt{-1}, \tag{1}$$

where $\psi(t, x)$ represents the dependent variable. The terms a and b are the coefficients of nonlinearity and group velocity dispersion (GVD). The function $F(|\psi|^2)\psi$ is a real-valued function and is $k-$times continuously differentiable [3], so that

$$cF(|\psi|^2)\psi \in \sum_{m,n=1}^{\infty} C^k((-n, n)X(-m, m); \mathrm{I}\,R^2). \tag{2}$$

The BME gives a real description of the dynamics of optical solitons in generalized form. Therefore, the equation is of exceptional interest in the study of fiber optics. The equation come into sight in 2010 [3]. It was studied by a lot of authors. The equation is a generalized model of the NLSE studied in optical fiber. The dynamical system technique has been applied to study the BME with perturbation terms and Kerr law nonlinearity [6]. Jafari [7] have applied the first integral technique to study the BME. The Non-Kerr law solitons have been studied in [8]. In [9], Kohl studied the BME with perturbed terms using

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the soliton perturbation theory. The Lie symmetry analysis was used to solve BME in [9]. Ahmed [10] used the Adomian decomposition technique to solve the BME with Kerr law nonlinearity. Crutcher et al. [11] have given the one-soliton-modulated cosine solution to the BME with log law nonlinearity. Manafian [12] also used the Exp-function technique to study the BME with Kerr law nonlinearity. Zhou [13] applied the Jacobian elliptic equations expansion to study the BME.

In this work, the BME will be examined with three types of nonlinear fibers, namely; quadratic-cubic, Kerr Parabolic laws nonlinearities. The SGEM [15] will be employed to integrate the equation for each type of nonlinearities with the aim of deriving the optical solitons of the equation.

2. Analytic study

To study Eq. (1), we use the following transformation

$$\psi(t,x) = u(\xi)e^{i\phi(t,x)},\ \ \xi = K(x+vt), \tag{3}$$

where

$$\phi = -kx + \omega t + \theta, \tag{4}$$

where $\phi(t, x)$ represents the phase component, k is the frequency, ω represents the wave number, θ represents the phase constant, v is the velocity and K is the width of the soliton. Putting Eq. (3) into Eq. (1) and separating the into real and imaginary components, two equations are obtained. The imaginary component gives

$$v = 2akm, \tag{5}$$

and the real part gives

$$aK^2muu'' - \left(ak^2mu^2 + \omega\right)mu^2 + bF\left(u^2\right)u^2 - (1-m)a\ \ K^2m(u')^2 = 0. \tag{6}$$

3. Description of the Sine-Gordon expansion method

Consider the following Sine-Gordon equation

$$\psi_{tx} = \alpha \sin(\psi), \tag{7}$$

where α is a non-zero constant. We apply the transformation

$$u(t,x) = u(\xi),\ \ \xi = \eta(x+vt), \tag{8}$$

where v is the traveling wave velocity. Substituting Eq. (8) into Eq. (7)

$$u'' = \frac{\alpha}{v\eta^2}\sin(u(\xi)). \tag{9}$$

Eq. (9) can be simplified to give

$$\left[\left(\frac{u}{2}\right)'\right]^2 = \frac{\alpha}{v\eta^2}\sin^2\left[\frac{u(\xi)}{2}\right] + C, \tag{10}$$

where C is a constant of integration. By letting $C = 0$, $w(\xi) = \frac{u(\xi)}{2}$ and $f^2 = \frac{\alpha}{v\eta^2}$, Eq. (10) reduces to

$$w'(\xi)^2 = f^2\sin^2(w(\xi)), \tag{11}$$

and in a more simplified form gives

$$w'(\xi) = f\sin(w(\xi)). \tag{12}$$

Setting $f = 1$ in Eq. (12), we get

$$w'(\xi) = \sin(w(\xi)). \tag{13}$$

Eq. (13) has the following solutions

$$\sin(w(\xi)) = \operatorname{sech}(\xi)\ \text{or}\ \ \cos(w(\xi)) = \tanh(\xi), \tag{14}$$

and

$$\sin(w(\xi)) = i\operatorname{csch}(\xi)\ \text{or}\ \ \cos(w(\xi)) = \coth(\xi). \tag{15}$$

To obtain the solution of the nonlinear partial differential equation of the following form

$$P(\psi, \psi_t, \psi_x, \psi_{tt}, \psi_{xx}, \psi_{tx}, \ldots) = 0, \tag{16}$$