Optik 157 (2018) 484–491

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Original research article

Dark and combined optical solitons, and modulation instability analysis in dispersive metamaterial

Mustafa Inc [a], Aliyu Isa Aliyu [a,b,*], Abdullahi Yusuf [a,b], Dumitru Baleanu [c,d]

[a] Firat University, Science Faculty, Department of Mathematics, 23119 Elazig, Turkey
[b] Federal University Dutse, Science Faculty, Department of Mathematics, 7156 Jigawa, Nigeria
[c] Department of Mathematics, Cankaya University, Ankara, Turkey
[d] Institute of Space Sciences, Magurele, Romania

ARTICLE INFO

Article history:
Received 23 October 2017
Received in revised form 3 November 2017
Accepted 3 November 2017

Keywords:
Complex envelope function ansatz
Modulation instability
Soliton

ABSTRACT

This paper obtains the dark and dark-bright or combined optical solitons to the nonlinear schrödinger equation (NLSE) describing propagation in dispersive metamaterial in optical fibers. The integration algorithm is the complex envelope function ansatz. This naturally lead to some constraint conditions placed on the soliton parameters which must hold for the solitons to exist. The intensities and the nonlinear phase shifts of the solitons are reported. Furthermore, the modulation instability analysis (MI) is studied based on the standard linear-stability analysis and the MI gain spectrum is got. Numerical simulation of some obtained results are analyzed with interesting figures showing the physical meaning of the solutions.

1. Introduction

Metamaterials are those materials that are engineered to acquire certain properties which are not natural. The elementary materials are analogous in repeating patterns, at dimensions that are smaller than the wave lengths of the phenomena they influence. metamaterials are man-made artificial structures which exhibit uncommon properties, so far unavailable in naturally occurring materials, such as reversal of Doppler effect and Cherenkov effect, reversal of Snell's law, etc. [1]. One key attribute of Metamaterials is that they show negative refractive index, which is the reason why they are often referred to as negative index material, because of their simultaneous negative electric permittivity and negative magnetic permeability [1–6].

The recent success in the theory of solitons has resulted in the generalization of many nonlinear evolution equations that have very wide applications in the fields of physics and applied mathematics [7]. The nonlinear wave phenomena can be observed in various scientific fields, such as optical fibers, plasma physics, fluid dynamics, etc [8–10]. The exact solutions of these nonlinear models plays an important role in the study of nonlinear phenomena. The existence of solitary wave solutions implies a perfect balance between nonlinearity and dispersion, which usually requires rather specific conditions and can not be established in general [10]. Recently, efficient ansatz have been proposed to construct various types of solutions to NLSEs [11–14]. These solutions include bright, dark and combined solitons with interesting properties. We observe many

* Corresponding author at: Firat University, Science Faculty, Department of Mathematics, 23119 Elazig, Turkey.
E-mail addresses: minc@firat.edu.tr (M. Inc), aliyu.isa@fud.edu.ng (A.I. Aliyu), yusufabdullahi@fud.edu.ng (A. Yusuf), dumitru@cankaya.edu.tr (D. Baleanu).

https://doi.org/10.1016/j.ijleo.2017.11.032
0030-4026/

other new progresses in nonlinear optics [15–59]. The NLSE describing the propagation in dispersive metamaterial is given by [1,4]

$$\psi_x = -\frac{isgn(\beta_2)}{2}\psi_{tt} + \delta_3\psi_{ttt} + ivN^2\{|\psi|^2\psi + is_1(\psi|^2\psi)_t - \tau_r\psi(\psi|^2)_t\}, \tag{1}$$

In Eq. (1), x, t and ψ represents the displacement variable, time variable and field. The terms δ_3, N and s_1 are the order of the soliton, the third-order dispersion and the self-steepening. The term τ_r is associated to Raman gain while β_2 is group velocity dispersion (GVD) [4].

Third-order dispersion is relevant for pulses of femtosecond, If the GVD approach zero, it is possible to ignore it for optical pulses with width of 100 femtoseconds or higher and also for those with order 1 Watt and the GVD being away from zero [4,5]. Therefore, Eq. (1) can be written as

$$\psi_x = -i\mu\psi_{tt} + i\gamma\{|\psi|^2\psi + is_1(\psi|^2\psi)_t - \tau\psi(\psi|^2)_t\}, \tag{2}$$

where

$$\mu = \frac{isgn(\beta_2)}{2}, \quad \tau = \tau_r, \quad \gamma = vN^2. \tag{3}$$

In [4], the optical chirped solitons of Eq. (2) are investigated using a nonlinear chirp expression.

In this work, we aim to investigate the existence of combined optical solitons of Eq. (2) using the complex envelope function ansatz [11,12,14] and the MI using the standard linear-stability analysis [8,15–17]. The stability gain spectrum of the equation will be investigated.

2. Complex envelope function ansatz

Assume a solution of the form [11,14]:

$$\psi(x,t) = A(x,t)e^{i\phi(x,t)}, \quad \phi(x,t) = -kx + \omega t + \theta, \tag{4}$$

where $A(x,t)$ is the complex envelope function and ϕ is the linear phase shift. The parameters k and ω in the phase component represent the wave number and wave frequency of the light pulse, respectively, while θ is the phase constant. The complex envelope ansatz [11] requires a modification of the solution in the form

$$A(x,t) = \mathrm{i}\beta + \lambda\tanh[\eta(x-vt)] + \mathrm{i}\rho\,\mathrm{sech}[\eta(x-vt)], \tag{5}$$

where η and v are the pulse width and velocity, respectively. In the case where $\beta = \lambda = 0$ in Eq. (5), we acquire bright soliton [11,14]. But when $\rho = 0$, the solution Eq. (5) transforms a dark soliton solution. When the parameters β, λ, ρ are non zero, the ansatz Eq. (5) describe the features of a dark-bright or combined soliton. If $\beta = 0$ and $\lambda, \rho \neq 0$, it implies that it is not possible to get a soliton with a pronounced platform underneath it. The accordingly envelope solution will describe the evolution of a combined soliton on a zero background having a constant amplitude and intensity. The parameters η, v, k, ω are real values, but the parameters λ, β, ρ can be real or complex numbers depending on the model parameters μ, γ, τ, s_1. The amplitude of $A(x,t)$ reads

$$|A(x,t)| = \left\{\lambda^2 + \beta^2 + 2\beta\rho\,\mathrm{sech}[\eta(x-vt)] + (\rho^2 - \lambda^2)\mathrm{sech}^2[\eta(x-vt)]\right\}^{\frac{1}{2}}. \tag{6}$$

In addition, the nonlinear phase shift ϕ_{NL} is given by

$$\phi_{NL} = \arctan\left[\frac{\beta + \rho\,\mathrm{sech}[\eta(x-vt)]}{\lambda\tanh[\eta(x-vt)]}\right]. \tag{7}$$

2.1. Application to Eq. (2)

We begin by substituting Eq. (4) into Eq. (2) to obtain

$$A\left(k + \mu\omega^2 + \gamma\left|A\right|\left(\left|A\right|(1-\omega s_1) - 2(\tau - is_1)(|A|)_t\right)\right) + i\left(A_x + \left(-2\mu\omega + \gamma\left|A\right|^2 s_1\right)A_t + i\mu A_{tt}\right) = 0. \tag{8}$$