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Refinement of the theory of formation of rainbow with incorporation of the refined unambiguous angles of incidence, reflection, and refraction

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ABSTRACT

This paper is concerned with one of the most common atmospheric optical phenomenon – the rainbow. In order to get rid of the ambiguity present in the traditional theory of formation of rainbow which is based on the traditional ambiguous angles of incidence, reflection, and refraction, a novel unambiguous theory of formation of rainbow which makes use of the refined unambiguous angles of incidence, reflection, and refraction has been offered. As a result, the present contribution will enhance and sophisticate the relevant optical physics literature there by enriching the same as well.

1. Introduction

Launching of the new world of geometrical optics [2], which is exclusively based on the refined unambiguous angles of incidence, reflection, and refraction, demands that all theoretical discussions or derivations of the traditional geometrical optics must proceed in the light of the refined unambiguous angles of incidence, reflection, and refraction instead of making use of the traditional ambiguous angles of incidence, reflection, and refraction to enhance and sophisticate the relevant optical physics literature as well as to bring preciseness in the relevant field of study. With that point in mind, the deviation problems in ray optics and the dispersive power of a prism have been considered in detail in [9] and [10] respectively in the light of the refined unambiguous angles of incidence, reflection, and refraction to obtain novel interesting results.

This paper considers the theory of formation of rainbow, which has been dealt with in the traditional literature [3–8] on the basis of the long-running ambiguous angles of incidence, reflection, and refraction, as a result of which such a theoretical treatment is not at all free from ambiguity as well. With a view to getting rid of the ambiguity present in the traditional theory of formation of rainbow, an unambiguous theory of formation of rainbow has been offered in this paper with the incorporation of the refined unambiguous angles of incidence, reflection, and refraction for the development of: (i) novel expression for the net deviation suffered by an incident ray in emerging out of a raindrop, and (ii) novel expression for the primary angle of incidence for the minimum deviation to occur, in terms of the refractive index of water and the total number of total internal reflections taking place inside the raindrop. As a result, the present contribution will bring preciseness and sophistication in the relevant field of the optical physics literature there by enhancing and enriching the same as well.

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2. Definitions and preliminaries

From the view point of the interest of readership, the following three definitions of the refined unambiguous angles of incidence, reflection, and refraction are being reproduced below from [2].

Refined unambiguous angle of incidence (i): The angle of incidence (i) is the smaller of the angles between the vectors **i** and **n** subject to the condition that $\pi/2 < i \le \pi$, so long as the case considered is a reflection (or a refraction of light as it passes from a rarer to a denser medium). If however it is a case of refraction as light passes from a denser medium to a rarer medium, the angle i must be bounded by the relation $0 \le i < \pi/2$.

Refined unambiguous angle of reflection (r): The angle of reflection (r) is the smaller of the angles between the vectors **r** and **n** subject to the condition that $0 \le r < \pi/2$.

Refined unambiguous angle of refraction (R): The angle of refraction (R) is the smaller of the angles between the vectors **n** and **R** subject to the condition that, $\pi/2 < R \le \pi$ when the ray of light passes from a rarer medium to a denser medium, or $0 \le R < \pi/2$ when the ray of light passes from a denser medium to a rarer medium.

Furthermore, in order to enhance the readability of the paper the two laws reported in [1] are also being reproduced below.

The generalized vectorial law of reflection: If **i** and **r** represent unit vectors along the directions of incident ray and reflected ray respectively and if **n** represents unit vector along the direction of the positive unit normal to the reflector at the point of incidence then

$$\boldsymbol{n}\times\boldsymbol{i} = \boldsymbol{n}\times\boldsymbol{r}$$

The generalized vectorial law of refraction: If **i** and **R** represent unit vectors along the directions of the incident ray and refracted ray of particular colour respectively and if **n** represents unit vector along the direction of the positive unit normal to the surface of separation at the point of incidence then

$$\boldsymbol{n}\times\boldsymbol{i} = \mu(\boldsymbol{n}\times\boldsymbol{R})$$

where μ = Refractive index of the second optical medium with respect to the first optical medium for the particular colour of light under consideration.

3. The novel unambiguous theory of formation of rainbow

Let us consider Fig. 1, in which the path of a ray of light incident on spherical raindrop and undergoing one total internal reflection is shown. O is the centre of the raindrop. The ray SA is incident on the surface of the raindrop at the point A. Then it gets refracted inside the drop along AB, undergoes total internal reflection at the point B and moves along BC, the points B and C lying on the surface of the raindrop. The positive unit normal vectors to the surface of the raindrop at the points A, B, and C are n_1, n_2, and n_3 respectively. At the point C, the ray BC after undergoing refraction emerges along CD. The refined unambiguous angles of incidence and refraction at the point A are taken as i and r respectively. Similarly the refined unambiguous angles of incidence and reflection at the point B are respectively r_1 and r_2. Furthermore, at the point C, refined unambiguous angles of incidence and refraction are respectively r_1 and i_1.

Now, the deviation due to refraction at the point A = $r - i$.

Again the deviation due to total internal reflection at the point B = $r_2 - r_1$

Furthermore, the deviation again due to refraction at the point C = $i_1 - r_1$.

Thus the net deviation (ó) suffered by the incident ray in emerging out of the raindrop is given by,

$$ó = (r - i) + (r_2 - r_1) + (i_1 - r_1)$$

$$\text{or, } ó = -i + i_1 + r - 2r_1 + r_2 \tag{1}$$

Now, it can be readily seen from Fig. 1 that, we have,

$r_1 = \pi - r$, $r_2 = \pi - r_1 = \pi - (\pi - r) = r$, and from the principle of reversibility of light for refraction at the point A, it also follows that $i_1 = \pi - i$.

Hence from the relation (1), we have,

$$ó = -i + (\pi - i) + r - 2(\pi - r) + r$$

$$= -\pi - 2i + 4r$$

Thus the expression for the net deviation (ó) suffered by the incident ray in emerging out of the raindrop is given by,

$$ó = -\pi - 2i + 4r \tag{2}$$