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Simplest memristive system

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ABSTRACT

Many potential applications are developed towards the new memristive systems. However, these systems are too complicated to produce the real elements, the simplifications of memristive systems are interesting to our future applications. The simplest memristive system is modeled to make the real elements more practical in the electric circuit, which may confirm three fingerprints of memristor. These results verified by Matlab and Multisim simulations indicate that the proposed memristive system produces a pinched hysteresis loop for a periodic input. The voltage-current relation-ships are investigated for two parallel memristor circuits-a parallel memristor and capacitor circuit, and a parallel memristor and inductor circuit. It may be convenient and useful for our future application in the circuit.

1. Introduction

Memristor would have attracted a lot of attention for useful potential applications since the successful realization of HP memristor [1]. The memristor is described formally by quantity between electric charge variable and magnetic flux variable. It may be broadened to any kind of two-terminal electric elements called memristive systems [2], which has been extended to a lot of dynamical systems [3–9], and can be enlarged to memcapacitor and meminductor dynamical systems [2]. 15 different kinds of quasi-ideal memristive systems have generated to extend an ideal memristive system [10].

Three important characters are described to classify memristive system from the dynamical systems [11] in the following: (i) Hysteresis loop of memristive system is a Lissajous curve pinched at the origin; (ii) If the frequency of input periodic signal increases, then the hysteresis loop becomes smaller and shrinks continuously; (iii) If the frequency can tends to infinity, then the figure of the loop of memristive system tends to a straight line.

More novel functionalities and new concepts of memristive systems are discovered in different. The novel memristive systems have been investigated for the future potential applications [12–25]. While most of the researches have so far concentrated on the characters of memristive systems, very little is investigated about the simplifications of memristive systems. However, these memristive systems are too complicated to generate the real elements, the simplifications of memristive systems are meaningful for our future applications. The voltage-current relationships are investigated for two parallel memristor circuits-aparallel memristor and capacitor circuit, and a parallel memristor and inductor circuit. It may be useful for our future application in the circuit.

This work is presented as follows: in Section 2, the simplest memristive system are designed, and three unique fingerprints are investigated. Followed by, basic characteristics of memrisitive system are investigated by the multisim simulations in

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Section 3. The voltage-current relationships of both parallel MC and parallel ML circuits are given in Sections 4 and 5. Some conclusions are finally gained in Section 6.

2. The simplest memristive system

If u(t) and y(t) are the input signal and output signal of the dynamical system (2), and the variable x is the n-dimensional vector, then the following system is given by

$$\mathrm{y(t) = g(x, u, t)}, \quad \dot{x}(t) = \mathrm{f(x, u, t)}, \tag{1}$$

where, f and g are two functions of state variables x and u. If two variables u(t) and y(t) are the voltage and current respectively, then the relations are defined as a voltage-controlled memrisitive system or a flux-controlled memrisitive system [2].

If the generalized function f(x; u; t) is u(t), the generalized function g(x; u; t) is x(t), then the memrisitive system is defined by this relations

$$\mathrm{y(t) = x(t)u(t)}, \quad \dot{x} = u(t). \tag{2}$$

Remark 1: There are three reasons to name the memrisitive system (2) as the simplest memristive system. First, all the coefficients are 1 for the memrisitive system (2); second, there is only one term for two functions y(t) and x (t); third, the largest degree of the terms are 2 for the memrisitive system (2). Hence, the simplest memristive system is given in the mathematical equation of (2) for our design. The main work for this paper is in the simplest manner for denoting the memristive system for the future circuit applications.

Three characters [11] may be used to identify as a memrisitive system. Consider an external sinusoidal stimulus as the input signal across the memrisitive system (2), that is,

$$\mathrm{u\,(t) = Asin\,(\omega t)}\,, \tag{3}$$

where A and ω are the amplitude and radian frequency of the input signal (3), respectively. Therefore,

$$\mathrm{x(t)} = \int_{-\infty}^{t} u\,(\tau)\,d\tau = x_0 + \int_0^t \mathrm{Asin}(\omega \mathrm{t})d\tau = x_0 + \frac{A}{w}[1 - \cos(\omega \mathrm{t})], \tag{4}$$

where $x_0 = \int_{-\infty}^{t} u(\tau)d\tau$ is the original state.

Substituting Eqs. (3) and (4) into system (2), then it yields

$$\mathrm{y(t)} = \{x_0 + \frac{A}{\omega}[1 - \cos(\omega \mathrm{t})]\}\mathrm{Asin}(\omega t) = \{x_0 + \frac{A}{\omega}\}\mathrm{Asin}(\omega \mathrm{t}) - \frac{A^2}{2\omega}\sin(2\omega \mathrm{t}) \tag{5}$$

Hence, two component are included in the output signal, which is related to the original state of memrisitive system and both amplitude and frequency of the input signal (3).

(i) All the pinched hysteresis loops should be pinched at the origin to a periodical sinusoidal signal, even any amplitude of the signal in Fig. 1(a), even any frequency of the signal in Fig. 1(b), and even different initial states in Fig. 1(c).

Fig. 1(a) shows the input-output relationship obtained at the amplitudes 1, 2 and 3 for the initial state $x_0 = 1$ and the angular velocity w = 1, respectively. The input-output relationship is obtained at the angular velocities 1, 2 and 3 for the initial state $x_0 = 1$ and the amplitude A = 1 in Fig. 1(b), respectively. Fig. 1(c) shows the input-output relationship obtained at the initial states 1, 2 and 3 for the amplitude A = 1 and the angular velocity w = 1, respectively.

(ii) If the input periodic signal frequency increases from some critical frequencies, then the pinched hysteresis loop shrinks continuously in Fig. 1(b).

The pinched hysteresis loop is produced by two odd symmetric loops T and T' in Fig. 1. The area S and S*continuously filling the space between the input signal and the output signal is computed by

$$S = \oint_{T'} y(t)du(t), \quad S^* = \oint_{T*} y(t)du(t). \tag{6}$$

Substitute (3), (5) into (6), we get

$$\begin{aligned} \mathrm{S} &= \int_0^{\frac{\pi}{2w}} (x_0 + \frac{A}{w})\mathrm{Asin(wt)} - \frac{A^2}{2w}\mathrm{sin(2wt)}d\mathrm{Asin(wt)} = (x_0 + \frac{A}{w})\frac{A^2}{2}\mathrm{sin}^2\mathrm{(wt)} - \int_0^{\frac{\pi}{2w}} \frac{A^3}{w} sin2wtd(Asin(wt)) \\ &= (x_0 + \frac{A}{w})\frac{A^2}{2} + \int_0^{\frac{\pi}{2w}} \frac{A^3}{w} cos^2(wt)dcos(wt) = (x_0 + \frac{A}{w})\frac{A^2}{2} + \frac{A^3}{3}\frac{cos^3(wt)}{3} = \frac{A^2}{2}(x_0 + \frac{A}{3w}). \end{aligned} \tag{7}$$

The pinched hysteresis loop area may be related to the original value of the state variable in (7). The loop area should increase monotonically with the amplitude increase of the excitation signal in (7). The loop area should decrease monotonically with the frequency increase of the excitation signal in (7). The results can keep in the conformity with Chua's result [11] and the simulation results.