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Elliptic perfect optical vortices

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A B S T R A C T

We generalize the concept of perfect optical vortices, studying the elliptic perfect optical vortices (EPOVs), which also have diameters independent on the topological charge. A pure-phase diffractive optical element is proposed for efficient generation of such EPOV. Intensity of EPOV generated by this element is order of magnitude higher than that of the EPOV generated by the Fraunhofer diffraction of an elliptic Bessel beam. It is also higher than that of the EPOV, generated approximately by an elliptical axicon. We obtain exact analytical expressions for the orbital angular momentum (OAM) density and for the total OAM of the EPOV. These expressions show that the normalized OAM of the EPOV is fractional and it always exceeds the OAM of the conventional circular perfect optical vortex, which equals the topological charge. It allows continuous controlling of the OAM by changing the ellipticity. We show analytically that the OAM density is maximal on the smaller side of the EPOV and is minimal on its larger side. The ratio between the maximal and minimal OAM density equals the squared ratio between the ellipse diameters. Using the proposed element, EPOVs of several topological charges are generated experimentally using a spatial light modulator. We experimentally confirm the independence of their size on the topological charge, which is determined interferometrically. Such EPOVs can be used for movement of microscopic particles along an ellipse with acceleration, as well as for generation of OAM-entangled photons.

1. Introduction

In the paper [1], the concept of the perfect optical vortex (POV) has been introduced. The dark hollow radius of the POV does not depend on its topological charge. In addition, the intensity distribution of the POV has a shape of an infinitely thin light ring. Since then, many works have been dedicated to the perfect vortices.

In the same 2013 year, microparticle dynamics within a "perfect" vortex beam was analyzed [2], including vortices with fractional topological charge. Later, an improved technique for generating the perfect vortex based on the width-pulse approximation of Bessel function was proposed in [3]. In [4], a perfect vortex beam was treated as the Fourier transformation of a Bessel beam. Thus, POV were approximately generated by Fourier transforming of the Bessel-Gauss beams of different orders. It was shown that controlling the radial wave vector of a Bessel-Gauss beam allows tuning of the POV ring radius. In [5], perfect optical vortices with large topological charge up to 90 were generated by a digital micromirror device. For this purpose, there were used either the binary Lee holograms or the superpixel binary encoding technique. In [6], an approach

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was proposed for creating three-dimensional multifocal perfect vortices arrays by using a high numerical aperture objective. In [7], the POV was generated the same way as in [4], i.e. by using the Fourier transformation of the Bessel-Gauss beams. Since variation of the POV ring radius requires changing the focal length of the Fourier lens, in this paper a method was introduced to adjust the POV radius by simple varying the separation between the lens and the axicon. Using such vortices in spontaneous parametric down-conversion (SPDC), non-collinear interaction of photons has been studied. It appeared that the angular spectrum of the SPDC photons is independent of the orbital angular momentum (OAM) of the pump photons. In [8], the POV beams were generated and scattered through a rough surface. It was shown that the size of produced speckles is independent on the order of the POV and their Fourier transform gives the random non-diffracting fields. The obtained results may be useful in applications of POV for authentication in cryptography. In [9], POVs were used in plasmonic structured illumination microscopy to improve the efficiency of the excitation of plasmonic standing wave patterns. Using the perfect vortices, the imaging resolution of less than 200 nm was produced. In [10], POV beam with thin ring width and controllable ring diameter was used in a graphene-based optical sensor for single cells refractive index measurements. It is shown that the sensitivity of such sensor reaches 9.3×10^{-4} refractive index units. Paper [11] is devoted to in situ determining of the topological charge (including half-integer values) of a POV by using the phase shift method. In addition to the scalar perfect vortices, paper [12] introduces perfect vectorial vortex beams, which have intensity profile independent on the polarization order and the topological charge, and in addition have stable intensity profile and state of polarization upon propagation. In our paper [13], we considered three optical elements to generate perfect optical vortices: (i) an amplitude-phase element with a transmission function proportional to the Bessel function, (ii) an optimal phase element with a transmission equal to the sign of the Bessel function, and (iii) a spiral axicon. The doughnut intensity was shown to be highest when using an optimal phase element, while the commonly used spiral-axicon-aided diffraction ring was found to be twice as wide as when generated using two other elements.

So, in all the papers that we are aware of, there are studies of only circular perfect optical vortices. On the other hand, optical vortices are studied long ago and it is quite natural that along with conventional circular optical vortices there was an interest in their generalization. In particular, instead of circular vortices, vortices with elliptic shapes were proposed both in free space [14] and in different media [15–17]. The work [18] studies transformation of an optical vortex with an embedded ellipticity of some degree. The work [18] is a continuation of earlier papers with studying of the elliptic optical vortices [19], [20]. Paper [21] introduced an elliptic vortex Hermite-Gaussian beam with a fractional OAM. Some ellipticity was also introduced to the conventional circularly symmetric optical vortices by an annular ellipse aperture, in order to measure the topological charge [22]. Obviously, elliptical Bessel beams have the plane wave spectrum in a form of an infinitely thin ellipse. So, elliptic POV (EPOV) can be observed in the Fraunhofer diffraction zone of the elliptical Bessel beam. Such beams were studied in [23–25]. In [23], a narrow light ellipse has been obtained in the far field of the elliptical Bessel beam. For generation of this ellipse, the initial light field has a transmittance function of the axicon and spiral phase plate, but stretched along one of the Cartesian coordinates. In [24], Bessel functions with their arguments having elliptic locus were used to generate the mask, which was then recorded using holographic technique. In [25], diffraction-free beams with elliptic Bessel envelope were studied in periodic media, such as 1D photonic crystal. The papers [23,24] were devoted mainly to the experimental generation of the elliptic Bessel beams (and consequently narrow light ellipse in the far field), while the paper [25] was devoted to the propagation properties in the periodic media. Neither the OAM density nor the summary OAM of the generated elliptical vortices have been studied in these papers. Energetic efficiency of the proposed elements (in particularly, stretched axicon with the spiral phase plate) and the thickness of the obtained light ellipses have not been considered as well.

In this work, we bring together these two areas of research of optical vortices – perfect vortices and non-circular vortices. We generalize the concept of perfect optical vortices and consider a perfect optical vortex with an elliptic shape. We obtain analytical expression for the complex amplitude of the EPOV when it is generated by Fourier transformation of the elliptic Bessel beam, when it is bounded by an elliptic aperture. Using this expression, we estimate the thicknesses of the light ellipse on the horizontal and vertical axes. We obtain an exact analytical expression for the total (summary) OAM of the EPOV, as well as for the OAM density. The density is shown to be higher on the smaller side of the EPOV. Horizontal and vertical diameters of the EPOV are shown to be independent on the topological charge, as is the case with the conventional circular POV. We propose a phase optical element which generates in the far field a light ellipse with several times smaller thickness than an elliptical axicon with an elliptical spiral phase plate. Using such element, EPOVs of several topological charges are generated experimentally using a spatial light modulator. EPOVs can find application for movement of microscopic particles along an ellipse with acceleration, as well as for generation of OAM-entangled photons.

2. Fraunhofer diffraction of an infinite elliptical Bessel beam

We consider the Bessel beam, which is stretched along one of the Cartesian coordinates. The complex amplitude of such elliptical Bessel beam reads as

$$E_{\text{in}}(x, y) = J_m\left(\alpha\sqrt{\frac{x^2}{a^2} + \frac{y^2}{b^2}}\right) \exp\left[im\ \arctan\left(\frac{a}{b}\frac{y}{x}\right)\right], \tag{1}$$