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Exact solitons in optical metamaterials with quadratic-cubic nonlinearity using two integration approaches

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ABSTRACT

The aim of this paper is to present soliton solutions in optical metamaterials. The quadratic-cubic nonlinearity is considered. Two efficient algorithms that are the $\exp(-\Phi(\eta))$-expansion method and extended Jacobi's elliptic function expansion scheme are used to carry out the mathematical analysis. As a result, analytical dark soliton, singular soliton and periodic solutions are obtained.

1. Introduction

Metamaterials (MMs) are a class of new type of artificial synthetic materials with some extraordinary electromagnetic (EM) properties not existing in ordinary materials [1–4]. Recently, the study of optical solitons in MMs has attracted many researchers attention because of soliton theory in optical MMs is a very important and fascinating area of research in nonlinear optics. Zhou et al. [5] studied solitons in MMs with parabolic law nonlinearity. Xu et al. [6] reported Raman solitons in MMs having polynomial law non-linearity employing travelling wave hypothesis. Veljkovic et al. [7] studied super-sech soliton dynamics in MMs by collective variable approach. Triki et al. [8] investigated the MMs with Kerr law nonlinearity, and derived dipole soliton solutions by adopting the complex amplitude ansatz. Biswas et al. [9] obtained bright and dark solitons for MMs. Ebadi et al. [10] demonstrated the existence of solitons in MMs with Kerr law nonlinearity using $F-$expansion approach. In our most recent work [11], we analyzed the MMs with quadratic-cubic nonlinearity, and then acquired bright and singular optical soliton solutions by extended trial scheme approach.

The nonlinear Schrödinger's equation (NLSE) that governs the dynamics of soliton propagation through optical metamaterials with quadratic-cubic nonlinearity is studied in this paper, which in the dimensionless form is given by [9,12–15]

$$iq_t + aq_{xx} + \left(b_1|q| + b_2|q|^2\right)q = i\left\{\alpha q_x + \beta\left(|q|^2 q\right)_x + \nu\left(|q|^2\right)_x q\right\} + \theta_1\left(|q|^2 q\right)_{xx} + \theta_2|q|^2 q_{xx} + \theta_3 q^2 q^*_{xx}. \tag{1}$$

In this model, the complex valued dependent variable that represents the wave envelope is denoted by q and its complex conjugate is q^*. The independent variables are x and t which respectively represent the spatial and temporal variables. The parameter a is the group velocity dispersion (GVD) while b_1 and b_2 together comprise the quadratic-cubic nonlinearity. On the right hand side, β represents the self-steepening (SS) and ν is the nonlinear dispersion (ND), while α represents the inter-modal dispersion (IMD). Finally, the coefficients θ_l for $l = 1, 2, 3$ arise in the context of MMs [9,12–15].

This paper will perform the $\exp(-\Phi(\eta))$-expansion approach [17–20] and extended Jacobi's elliptic function expansion scheme [21–25] to obtain analytical solutions to the NLSE with quadratic-cubic nonlinearity. Details will be shown in the next sections.

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2. Soliton solutions

To derive soliton solutions to the governing equation, the starting hypothesis is [9,13,16]

$$q(x,t) = P(\eta)\exp[i\phi(x,t)], \tag{2}$$

where

$$\eta = k(x - vt), \tag{3}$$

and the phase component ϕ is given by

$$\phi(x,t) = -\kappa x + \omega t + \theta. \tag{4}$$

In (2) and (3), $P(x, t)$ represents the amplitude portion of the soliton, and k and v are inverse width and velocity of soliton. The parameters κ and ω in (4) represent the frequency and wave number of the soliton, respectively while θ is the phase constant. Substituting (2) into (1), one obtains a pair of relations. Imaginary part gives

$$v = -\alpha - 2a\kappa, \tag{5}$$

and

$$3\beta + 2v - 2\kappa(3\theta_1 + \theta_2 - \theta_3) = 0, \tag{6}$$

while real part leads to

$$\begin{aligned} &ak^2P'' - \left(\omega + a\kappa^2 + \alpha\kappa\right)P + b_1P^2 + \left(b_2 - \beta\kappa + \kappa^2\theta_1 + \kappa^2\theta_2 + \kappa^2\theta_3\right)P^3 \\ &\quad - k^2\left(3\theta_1 + \theta_2 + \theta_3\right)P^2P'' - 6k^2\theta_1P(P')^2 = 0. \end{aligned} \tag{7}$$

In order to extract an analytic solution, we apply the transformations $\theta_1 = 0$ and $\theta_2 = -\theta_3$ in Eq. (7) to find

$$ak^2P'' - \left(\omega + a\kappa^2 + \alpha\kappa\right)P + b_1P^2 + (b_2 - \kappa\beta)P^3 = 0, \tag{8}$$

where

$$3\beta + 2v + 4\kappa\theta_3 = 0. \tag{9}$$

The $\exp(-\Phi(\eta))$-expansion approach and extended Jacobi's elliptic function expansion scheme will now be applied, in the following sections, to Eq. (8) to obtain bright, dark and singular soliton solutions to (1).

3. $\exp(-\Phi(\eta))$-Expansion approach

To start off with $\exp(-\Phi(\eta))$-expansion approach, the initial assumption of the solution structure of (8) is taken to be:

$$P(\eta) = \sum_{i=1}^{N} A_i(\exp[-\Phi(\eta)])^i, \tag{10}$$

where A_i for $i = 0, 1, \ldots, N$ are constants to be determined later, such that $A_N \neq 0$, while the function $\Phi(\eta)$ is the solution of the auxiliary ordinary differential equation (ODE)

$$\Phi'(\eta) = \exp[-\Phi(\eta)] + \mu \ \exp[\Phi(\eta)] + \lambda. \tag{11}$$

It is well known that Eq. (11) has solutions in the following forms:

If $\mu \neq 0$ and $\lambda^2 - 4\mu > 0$,

$$\Phi(\eta) = \ln\left(-\frac{\sqrt{\lambda^2 - 4\mu}\tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}(\eta + C)\right) + \lambda}{2\mu}\right). \tag{12}$$

For $\mu \neq 0$ and $\lambda^2 - 4\mu < 0$,

$$\Phi(\eta) = \ln\left(\frac{\sqrt{4\mu - \lambda^2}\tan\left(\frac{\sqrt{4\mu-\lambda^2}}{2}(\eta + C)\right) - \lambda}{2\mu}\right). \tag{13}$$

However, when $\mu = 0$, $\lambda \neq 0$ and $\lambda^2 - 4\mu > 0$,

$$\Phi(\eta) = -\ln\left(\frac{\lambda}{\exp(\lambda(\eta + C)) - 1}\right). \tag{14}$$