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Evanescent spherical field of charged particle at rest

Alojz Suhadolnik

University of Ljubljana, Faculty of Mechanical Engineering, Aškerčeva 6, 1000 Ljubljana, Slovenia

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ABSTRACT

An exterior of a charged particle at rest can be well described with evanescent fields. In the paper, an electric scalar monopole potential and magnetic scalar dipole potential have been calculated on the base of an evanescent scalar monopole field and evanescent scalar dipole field. It has been shown that in a zero frequency limit only the evanescent fields surround the particle at rest. A similar situation can be achieved by approaching the radius of the source to the zero. In this case, the charged particle at rest can be viewed as a dynamic object.

1. Introduction

A spherical charged particle exterior can be well described with an evanescent spherical field around a spherical core. A wave differential equation enables the calculation regarding the dynamic behavior of the particle [1]. If a wave number is set to zero, the created static field around the sphere with a wave source in the center can be well described using a Poisson's differential equation. Approaching the zero frequency limit the evanescent waves take the dominant role while the propagating waves disappear. This situation is of great interest due to a power law decay of the evanescent field in this limit. A similar situation can be achieved by approaching the radius of the source to the zero. If the radius of the source is sufficiently small, the evanescent waves prevail and the charged particle does not radiate.

Several studies describe moving free electrons which also carry exponentially decaying evanescent field. These studies are mainly connected with the Čerenkov radiation [2,3] and Smith-Purcell radiation where the evanescent waves created around a moving particle are studied [4,5]. The evanescent waves are nowadays extensively involved in high-resolution microscopy techniques [6]. Even the amplification of the evanescent waves is described in the literature [7].

In this paper, the spherical evanescent waves around the charged particle at rest are described. In order to investigate the spherical waves approaching the zero frequency limit, the monopole and dipole spherical sources are presented. The corresponding fields are calculated using a multipole expansion including an angular spectrum representation technique. Later both the electric scalar monopole potential and magnetic scalar dipole potential are calculated and related to the evanescent fields of the monopole and dipole. In addition, the case of a small radius of the charged particle is investigated.

2. Spherical wave source

An inhomogeneous Helmholtz differential equation is derived from a wave equation with a Fourier transform [8]

$$(\nabla^2 + k^2)\Psi(\mathbf{r},\omega) = -f(\mathbf{r},\omega). \quad (1)$$

E-mail address: alojz.suhadolnik@fs.uni-lj.si

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In the equation $k = \omega/c$ is a wave number, c wave velocity, ω angular frequency, $\mathbf{r}$ vector of a space point and ∇^2 Laplacian operator. Furthermore, $\Psi = \Psi(\mathbf{r}, \omega)$ denotes a spatial and frequency dependent wave function and $f(\mathbf{r}, \omega)$ denotes a source function. In the zero frequency limit, k approaches to the zero $k \to 0$. Consequently when $k = 0$ the Helmholtz differential equation transforms to the Poisson's differential equation

$$\nabla^2 \Phi(\mathbf{r})^{k=0} = -C\rho(\mathbf{r}). \tag{2}$$

In this equation, the wave function is replaced by a scalar potential function $\Phi(\mathbf{r})$ and the source function with a charge density function $\rho(\mathbf{r})$ multiplied with a constant C.

The charge density function describes a charge distribution of the sphere. The formal solution of the Poisson's equation can be written in an integral form over the volume V' and consequently a scalar potential is deduced, where vector $\mathbf{r}'$ defines a source position

$$\Phi(\mathbf{r})^{k=0} = \frac{C}{4\pi} \int \frac{1}{|\mathbf{r} - \mathbf{r}'|} \rho(\mathbf{r}') dV'. \tag{3}$$

The function under the integral $1/(4\pi|\mathbf{r} - \mathbf{r}'|)$ is called Green's function. It is defined in a static case $k = 0$ as a multipole expansion. For $r > r'$ this expansion is [8]

$$\frac{1}{4\pi|\mathbf{r} - \mathbf{r}'|} = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \frac{1}{2l+1} \frac{r'^l}{r^{l+1}} Y_l^m(\theta, \phi) Y_l^{m*}(\theta', \phi'), \; r > r'. \tag{4}$$

In the multipole expansion, the term $Y_l^n(\theta, \phi)$ and its complex conjugate $Y_l^{n*}(\theta', \phi')$ are angular functions and represent spherical harmonics where l and n are integers. The range of l is from the zero to infinity while n occupies an interval $-l$...., 0, ... $+l$. The spherical harmonics in spherical coordinates (r, θ, ϕ) have the following form [9]

$$Y_l^n(\theta, \phi) = \sqrt{\frac{2l+1}{4\pi}} \sqrt{\frac{(l-n)!}{(l+n)!}} P_l^n(\cos\theta) e^{in\phi}. \tag{5}$$

In this equation $P_l^n(\cos\theta)$ denotes associated Legendre polynomials. Inserting multipole expansion (4) into the solution of the Poisson's equation (3) gives another expression of the scalar potential

$$\Phi(r, \theta, \phi)^{k=0} = C \sum_{l=0}^{\infty} \sum_{n=-l}^{l} \frac{1}{2l+1} \frac{Y_l^n(\theta, \phi)}{r^{l+1}} \int \rho(r') r'^l Y_l^{n*}(\theta', \phi') dV'. \tag{6}$$

3. Evanescent spherical scalar monopole field

Let a wave vector $\mathbf{k} = (k_x, k_y, k_z)$ be defined in a three-dimensional Cartesian coordinate system on the base of the wave number $k = \sqrt{k_x^2 + k_y^2 + k_z^2}$. A complete radial monopole spherical field of the outgoing waves can be expressed with [9–11]

$$\mathrm{G} = \frac{e^{ikr}}{r}. \tag{7}$$

In the zero frequency limit, this equation reduces to $\mathrm{G}^{k=0} = \lim_{k\to 0}(e^{ikr}/r) = 1/r$. In the limit, the evanescent spherical field $\mathrm{G}_e^{k=0}$ of the monopole can be calculated. The calculation involves an angular spectrum representation [12] for an upper hemisphere $z \geq 0$. Lower hemisphere $z < 0$ can be calculated similarly [13]

$$\mathrm{G}_e^{k=0} = \frac{1}{2\pi} \iint_{k_x^2+k_y^2 \geq 0} \frac{1}{\sqrt{k_x^2 + k_y^2}} e^{i(k_x x + k_y y) - \sqrt{k_x^2+k_y^2} z} dk_x \, dk_y = \frac{1}{r}, z \geq 0. \tag{8}$$

The propagating part $\mathrm{G}_p^{k=0}$ immediately follows from the expression $\mathrm{G}^{k=0} = \mathrm{G}_e^{k=0} + \mathrm{G}_p^{k=0}$ [13]

$$\mathrm{G}_p^{k=0} = \frac{1}{2\pi} \iint_{k_x^2+k_y^2 \leq 0} \frac{1}{\sqrt{k_x^2 + k_y^2}} e^{i(k_x x + k_y y) - \sqrt{k_x^2+k_y^2} z} dk_x \, dk_y = 0, \; z \geq 0. \tag{9}$$

This means that the propagating field of the monopole vanishes in the zero frequency limit while the evanescent field of the monopole has the power law decay and is equal the complete radial monopole field $\mathrm{G}_p^{k=0} = 0 \Rightarrow \mathrm{G}^{k=0} = \mathrm{G}_e^{k=0}$.

4. Evanescent spherical scalar dipole field

The partial derivative of the scalar monopole field $\mathrm{G} = e^{ikr}/r$ in z direction determines a dipole field [14–17]

$$\dot{\mathrm{G}} = \frac{\partial \mathrm{G}}{\partial z} = \frac{z e^{ikr}(ikr - 1)}{r^3}, \tag{10}$$