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Resonant optical soliton perturbation with anti-cubic nonlinearity by extended trial function method

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ABSTRACT

This paper studies the resonant optical solitons that comes with anti-cubic nonlinearity in presence of Hamiltonian perturbation terms. The extended trial function method is employed to retrieve cnoidal waves, bright and singular soliton solutions as well as periodic singular solutions to the model.

1. Introduction

Optical soliton perturbation is a treasure-trove in the field of mathematical photonics. This area of research has a long standing history ever since soliton propagation through optical fibers has been proposed for all-optical communications. There has been a constant visibility of novel results in various journals across the globe [1–15]. This paper studies the perturbed resonant soliton solution in an anti-cubic (AC) nonlinear medium. The governing equation is the nonlinear Schrödinger's equation (NLSE) along with a few Hamiltonian perturbations. This NLSE with AC nonlinearity was first proposed and studied in 2003 [9]. Later, this nonlinearity was studied extensively by various authors and a flood of results are reported over the years. In the past, resonant optical solitons with AC nonlinearity was studied with the unperturbed NLSE [2,5]. Additionally, optical solitons with AC nonlinearity was studied using the semi-inverse-variational principle [4], method of undetermined coefficients, tanh–coth method and simple equation algorithm [11]. The soliton perturbation the-

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ory for AC nonlinear medium was also addressed during 2017 [3]. This paper focuses on the application of the extended trial equation method to retrieve soliton solutions to the perturbed resonant NLSE with AC nonlinearity. These results will thus be an extension or generalization of the earlier reported results of the unperturbed version. This integration tool was successfully applied earlier to magneto-optic waveguides and DWDM systems [6,7]. This paper will detail the application of the algorithm to perturbed resonant NLSE with AC nonlinearity.

1.1. Mathematical model

The governing resonant NLSE, for AC nonlinearity, with perturbation terms that is studied in nonlinear optics is given in its dimensionless form as [2–4]:

$$iq_t + aq_{xx} + b\frac{|q|_{xx}}{|q|}q + \left(\frac{c_1}{|q|^4} + c_2|q|^2 + c_3|q|^4\right)q = i\left\{\alpha q_x + \lambda\left(|q|^2 q\right)_x + \nu\left(|q|^2\right)_x q + \theta|q|^2 q_x\right\} + \sigma\frac{q^*_{xx}}{|q|^2}q^2. \tag{1}$$

In (1), x and t are spatial and temporal variables respectively and $q(x, t)$ is the complex-valued dependent variable. Here, $i = \sqrt{-1}$ and the first term on the left side is the temporal evolution. The coefficient of group velocity dispersion is a while b is the coefficient of Bohm potential for chiral solitons with quantum Hall effect. The nonlinearities stem out from the coefficients of c_j, for $j = 1, 2, 3$. In particular, c_1 gives the effect of AC nonlinearity. When $c_1 = 0$, it is parabolic law nonlinearity that kicks in. On the right hand side of (1), α is the inter-modal dispersion, and λ is the self-steepening term for short pulses while ν and θ represents nonlinear dispersion. Finally, σ represents relativistic effect in plasmas. This is the structure of (1) and it will be addressed by extended trial function method (ETFM) [4–9] in the subsequent section.

2. Mathematical analysis

In order to solve Eq. (1) by ETFM, the starting assumption is:

$$q(x, t) = g(\xi)e^{i\phi(x,t)} \tag{2}$$

where $g(\xi)$ represents the shape of the pulse and

$$\xi = \mu\left(x - vt\right), \tag{3}$$

and the phase component is defined as

$$\phi(x, t) = -\kappa x + \omega t + \theta_0. \tag{4}$$

Here κ is the soliton frequency, ω is the wave number of the soliton and θ_0 is the phase constant. Also, in (3), v represents the speed of the soliton. Substituting (2) into (1) and decomposing into real and imaginary parts, give

$$\mu^2\left(a + b - \sigma\right)g'' - (\omega + \alpha\kappa + a\kappa^2 - \sigma\kappa^2)g + \left\{\left(c_2 - \lambda\kappa - \theta\kappa\right)g^2 + \frac{c_1}{g^4} + c_3 g^4\right\}g = 0, \tag{5}$$

and

$$v + \alpha + 2a\kappa + 2\sigma\kappa + \left\{3\lambda + 2\nu + \theta\right\}g^2 = 0. \tag{6}$$

Thus, Eq. (6) gives the speed of the soliton as:

$$v = -\alpha - 2(a + \sigma)\kappa, \tag{7}$$

along with the constraint condition on the perturbation parameters:

$$3\lambda + 2\nu + \theta = 0. \tag{8}$$

In order to proceed, multiply both sides of Eq. (5) by g' and integrating with respect to ξ, we get

$$\mu^2\left(a + b - \sigma\right)\left(g'\right)^2 - (\omega + \alpha\kappa + a\kappa^2 - \sigma\kappa^2)g^2 + \frac{1}{2}\left(c_2 - \lambda\kappa - \theta\kappa\right)g^4 - \frac{c_1}{g^2} + \frac{c_3}{3}g^6 + c_4 = 0, \tag{9}$$

where c_4 is a constant of integration, to be determined by model parameters.

Introducing the substitution,

$$g^2 = U, \tag{10}$$

Eq. (9) simplifies to:

$$\mu^2\left(a + b - \sigma\right)\left(U'\right)^2 - 4(\omega + \alpha\kappa + a\kappa^2 - \sigma\kappa^2)U^2 + 2\left(c_2 - \lambda\kappa - \theta\kappa\right)U^3 - 4c_1 + \frac{4c_3}{3}U^4 + 4c_4 U = 0. \tag{11}$$

Eq. (11) will now be analyzed by ETFM to derive soliton and other solutions to the model (1).