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Analysis of a four-wing fractional-order chaotic system via frequency-domain and time-domain approaches and circuit implementation for secure communication☆

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ABSTRACT

This paper firstly discusses chaotic behaviors in a four-wing fractional-order system based on frequency-domain approach and time-domain approach, respectively. And chaotic dynamics is indeed found to exist in the system by numerical analysis. Then, in order to apply the system in secure communication, a new chaos circuit based on the four-wing fractional-order chaotic systems is designed to implement the synchronization mentioned. And the effectiveness and feasibility of the proposed synchronization scheme are verified by the new analog circuit. At last, an example of secure communication is given to show that the fractional-order four-wing system is appropriate for secure communication of some signals.

1. Introduction

In recent years, the study on theory and application of fractional-order chaos has been becoming a new hot spot. The research regarding fractional-order systems includes finding chaotic dynamics by numerical analysis [1–6], giving the stability analysis [7–9], building the analog circuits to implement [10–14], designing methods of chaos synchronization and encryption schemes for secure communication [15–20], and so on [21–27]. It is very difficult to directly compute the solutions of fractional-order systems compared to computing integer-order systems. Therefore, approximation methods are always adopted to analyze their chaotic dynamics in many papers. At the present, there are mainly two kinds of approximation methods, one is frequency-domain approach used in [11–14,28–33], and the other is time-domain approach used in [20,21,27,34–36]. Generally speaking, the former is more convenient to design an analog circuit to implement a fractional-order system from viewpoint of application [11–13]. And the latter is more sufficient and reliable to explain chaotic behaviors [27,34–39]. In short, if a fractional-order system is both found to show chaotic dynamics by adopting two different approximation methods, one can conclude that this system is chaotic. In addition, chaos synchronization, which is commonly applied in secure communication, has attracted great interest of many researchers from engineering application of point of view. Since PC chaotic synchronization was proposed in 1990, the chaos synchronization, especially for fractional-order

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systems, has been booming [17,40–45]. However, most of the reported papers on fractional-order chaos synchronization are just numerical analysis, which seems inpractical for fractional-order chaos synchronization to be applied in secure communication.

Recently, based on frequency-domain approach, a four-wing fractional-order system was reported to be chaotic, and some proofs including numerical analysis and analog circuits were given to verify its chaotic characteristics [12]. In order to show the chaotic characteristic of the four-wing fractional-order system adequately, this paper further discusses chaotic behaviors of the four-wing fractional-order system investigated in [12] based on both frequency-domain approach and time-domain approach, respectively. It is very interesting that chaotic behaviors are both found when utilizing the above two approaches, that is, chaos is indeed found to exist in the system by numerical analysis, no matter wether frequency-domain approach or time-domain approach. What is more interesting is that some results from the above two approaches are very similar, which will attract more attention for further studying, analyzing and utilizing the system. Therefore, the paper further designs a new analog circuit to implement the synchronization for the fractional-order system. The results from circuit experiments verify the effectiveness of the proposed synchronization method in [46]. At last, based on the above fractional-order synchronization analog circuit, a secure communication circuit is also designed to study secure communication.

2. A four-wing fractional-order chaotic system

A four-wing fractional-order system is described as [12],

$$\begin{cases} \dfrac{d^{\alpha 1}x}{dt^{\alpha 1}} = f_1(x,y,z) = ax + ky - yz \\ \dfrac{d^{\alpha 2}y}{dt^{\alpha 2}} = f_2(x,y,z) = -by - z + xz \\ \dfrac{d^{\alpha 3}z}{dt^{\alpha 3}} = f_3(x,y,z) = -x - cz + xy \end{cases} \tag{1}$$

where system parameters a, b, c, $k \in R$, and fractional order $0 < \alpha_1 < 1$, $0 < \alpha_2 < 1$, $0 < \alpha_3 < 1$. In [12], some results including bifurcation diagrams, phase portraits, topological horseshoe, and circuit implementation, have been reported to prove the dynamics of the system (1). However, the Lyapunov exponents of system (1) were not given to show chaotic dynamics. The positive Lyapunov exponent is an important feature to determine chaotic dynamics. Therefore, Lyapunov exponents diagram of the fractional-order system (1) will be reported in this paper.

In this paper, the frequency-domain approach is based on [28]. The approximation functions with an error of 2 dB are listed as follows:

$$\frac{1}{s^{0.9}} \approx \frac{1.766s^2 + 38.27s + 4.914}{s^3 + 36.15s^2 + 7.789s + 0.01} \tag{2}$$

$$\frac{1}{s^{0.8}} \approx \frac{5.235s^3 + 1453s^2 + 5306s + 254.9}{s^4 + 658.1s^3 + 5700s^2 + 658.2s + 1} \tag{3}$$

$$\frac{1}{s^{0.7}} \approx \frac{5.406s^4 + 177.6s^3 + 209.6s^2 + 9.179s + 0.0145}{s^5 + 88.12s^4 + 279.2s^3 + 33.3s^2 + 1.927s + 0.0002276} \tag{4}$$

$$\frac{1}{s^{0.6}} \approx \frac{8.579s^4 + 255.6s^3 + 405.3s^2 + 35.93s + 0.1696}{s^5 + 94.22s^4 + 472.9s^3 + 134.8s^2 + 2.639s + 0.009882} \tag{5}$$

And the adopted time-domain approach is the advised Adams–Bashforth–Moulton method [29,22], and thus for the initial values x_0, y_0, z_0 and h step length, on the basis of the above method, the fractional-order four-wing system (1) can be written as following discrete form:

$$\begin{cases} x_{n+1} = x_0 + \dfrac{h^{\alpha_1}}{\Gamma(\alpha_1+2)}[f_1(x_{n+1}^p, y_{n+1}^p, z_{n+1}^p) + \sum\limits_{j=0}^{n} \beta_{1,j,n+1} f_1(x_j, y_j, z_j)] \\ y_{n+1} = y_0 + \dfrac{h^{\alpha_2}}{\Gamma(\alpha_2+2)}[f_2(x_{n+1}^p, y_{n+1}^p, z_{n+1}^p) + \sum\limits_{j=0}^{n} \beta_{2,j,n+1} f_1(x_j, y_j, z_j)] \\ z_{n+1} = z_0 + \dfrac{h^{\alpha_3}}{\Gamma(\alpha_3+2)}[f_2(x_{n+1}^p, y_{n+1}^p, z_{n+1}^p) + \sum\limits_{j=0}^{n} \beta_{3,j,n+1} f_1(x_j, y_j, z_j)] \end{cases} \tag{6}$$